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DUMMETT'S FREGEAN SOLUTION TO THE PARADOX OF ANALYSIS

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Abstract: The main topic of the essay is a puzzle over a certain kind of identity statements. There are identity statements that claim to be in the same time true and informative. This is puzzling because in order for the identity to be true, the two sides must be identical and in order to be informative the two sides must be different. These kinds of identity statements are not unusual (see any mathematical equation) but they are difficult to explain. I will focus o a Fregean kind of explanation and I will critically analyze Dummett's take on a Fregean solution to the paradox of analysis.

Keywords: paradox of analysis, Frege, Dummett, sense, reference, identity statements.

The paradox of analysis places under scrutiny definitions, analyses, explanations, i.e. statements that may be subsumed under the general heading 'identity statements'. The dilemma stems from (apparently) conflicting requirements: in order to be correct, the right and the left side of the identity statement should have something in common; on the other hand, in order to be non-trivial, the two sides of the identity must also incorporate significant differences.

The paradox of analysis appears under this specific name especially in connection with G.E. Moore, namely when C. H. Langford questions Moore's notion and method of analysis:

> It is indeed possible to deny that analysis can be a significant philosophical or logical procedure. This is possible, in particular, on the ground of the so-called paradox of analysis, which may be formulated as follows. Let us call what is to be analyzed the analysandum, and let us call that which does the analyzing, the analysans. The analysis then states an appropriate relation of equivalence between analysandum and the analysans. And the paradox of analysis is to the effect that, if the verbal expression representing the analysandum has the same meaning as the verbal expression representing the analysans, the analysis states a bare identity and is trivial; but if the two verbal expressions do not have the same meaning, the analysis is incorrect. [1]

[1] C.H. Langford, "Moore's Notion of Analysis", p. 322.

Frege's own formulation of the paradox of analysis can be found in his review to Edmund Husserl's *Philosophie der Arithmetik*[2], where the dilemma is presented as a Husserlian objection:

> If words and combinations of words mean ideas, then for any two of them there are only two possibilities: either they designate the same idea or they designate different ideas. In the former case, it is pointless to equate them by means of a definition: this is an 'obvious circle'; in the latter case is wrong. These are also the objections the author raises [i.e. Husserl], one of them regularly. A definition is also incapable of analyzing the sense, for the analyzed sense is just not the original one. In using the word to be explained, I either think clearly everything I think when I use the defining expression: we then have the 'obvious circle'; or the defining expression has a more richly articulated sense, in which case, I do not think the same thing in using it as I do in using the word to be explained: the definition is then wrong.[3]

The problem, therefore, appears when we try to establish *which* part has to be the *same* and *which* part should be *different* in order to consider some piece of information as genuinely new. Consequently, the problem of what stays the same and what does not - in a Fregean framework - becomes a problem of choosing between a solution of the type 'same sense-same reference' and a solution of the type 'different sense-same reference'. In other words, a possible answer to the question "can an analysis be both correct and informative?" will have to frame an informative correct analysis either as a 'same sense-same reference' type of phrase or as a 'different sense-same reference' type of phrase. The main question becomes, at this point, the following: is it possible to have genuine novelty in a 'same sense-same reference' type of situation? It is obvious that the novelty is possible in the other type of situation discussed, namely 'different sense – same reference'. To use Frege's example, an identity like 'Morning Star = Evening Star' offers a ready explanation for its informativeness: it is a different sense-same reference type of situation. But Dummett claims that novelty may appear even in same sense-same reference type of situation.

Michael Dummett's answer to the paradox of analysis involves a 'same sense-same reference' type of solution, i.e. he claims that it is possible to have new information in a 'same sense, same reference' type of situation.

His solution will be presented below together with my criticism and observations.

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[2] In 1891 Husserl's *Philosophie der Arithmetik* was published; several points from Frege's *Grundlagen* (1884) are criticized here, including the crucial definition of number by means of one-to-one correlation. Frege's reply comes in 1894, when his review of Husserl's *Philosophie der Arithmetik* was published in *Zeitschrift fur Philosophie und phil. Kritik,* vol. 103.

[3] Gottlob Frege, Review: Husserl, Philosophy of Arithmetic, in *Collected Papers on Mathematics, Logic and Philosophy*, p. 199.

1. MICHAEL DUMMETT ABOUT THE PARADOX OF ANALYSIS

Michael Dummett proposes an answer along Fregean lines and in terms of "pattern recognition". In *Frege: Philosophy of Mathematics,* he summarizes the situation in the following terms:

> Independently of whether mathematical truths are taken to be analytic or synthetic, mathematics compels us to recognize the fruitfulness of deductive inference; (...) that deductive reasoning can yield a vast range of unexpected consequences is therefore incontrovertible: the problem is how to explain this without rendering the validity of such reasoning problematic. It is tempting to explain the validity of simple inferential steps by appeal to the thesis that a knowledge of the premisses carries with it a knowledge of the conclusion. (...) If the thesis were true, we should already know all the consequences attainable by a sequence of such simple steps, however long; (...) As Frege remarked in § 88, the conclusions are contained in the premisses, not as rafters within a house, but as the plant within the seed.[4]

His solution combines *sameness* and *difference* by showing that the pattern we extract from an inferential process is acquired by us with the status of *new information*; as the pattern was also present in the sequence where it was extracted from, this constitutes the common ground between the old and the new context:

> The solution lies in drawing an appropriate distinction between form and content. All conceptual thought involves the apprehension of pattern: a report of current observation singles out particular features from a multifarious field of perception, subsuming them under general concepts. Some patterns force themselves upon us, but others need to be discerned. The characteristic of a pattern is that it is there to be discerned, but that, to apprehend that in which it is a pattern, we do not need to discern the pattern; it is essential to our discerning of the pattern that we recognize that that in which we have discerned it remains unaltered. One can hear a poem without identifying the metre or the rhyme scheme; someone unfamiliar with the Fibonacci sequence may fail to detect the principle determining the terms. When we become conscious of the metre or of the rule of generation, we perceive that pattern in the poem or the sequence, which we recognize as still the same poem or sequence as before[5].

In other words, Dummett's general solution relies on the fact that many times, when confronted with inferences, we fail to recognize the rule they obey, though this is also the rule we are using for making the inference; in other words, it relies on our *unawareness* of the instruments used in thinking. Dummett attributes this solution to Frege, which is not an unlikely tenet: "Frege's solution involved precisely the idea of discerning a pattern within a thought, or, rather, in the terminology of the early period, a judgeable content, a pattern is shared with a certain range of other thoughts

[4] Michael Dummett, *Frege: Philosophy of Mathematics*, pp. 36–37.
[5] *Ibidem*, p. 37.

or contents."[6] However, several objections (regarding how much of this idea can be said to have a place in a Fregean framework and how viable is this solution in general) could be raised against this tenet. They will be raised in the next sections.

From the previous passage, it becomes clear which is the solution proposed (leaving aside if it is really a Fregean solution): when we discover the rule "hidden" in one context (i.e. when we discern it), we acquire the new piece of information. Because the rule was all the time there, unnoticed, the validity of the process (i.e. preserving its correctness) is secured:

> Deductive reasoning is thus in no way a mechanical process, thought it may be set out so as to be checkable mechanically: it has a *creative component*, involving the apprehension of patterns within the thoughts expressed, and relating them to one another, that are not required for or given with a grasp of those, thoughts themselves. Since it has this creative component, a knowledge of the premisses of an inferential step does not entail a knowledge of the conclusion, even when we attend to them simultaneously; and so deductive reasoning can yield new knowledge. Since the relevant patterns need to be discerned, such reasoning is fruitful; but, since they are there to be discerned, its validity is not called in question. (...)
>
> It is surely that conception that can alone explain how such reasoning can be at the same time fruitful and cogent in virtue solely of the contents of the thoughts involved.[7]

The connection between Fregean framework, pattern recognition and formation of new concepts consists, according to Dummett, in the fact that the recognition of a pattern is done mainly by a process of decomposition or "dissection", a process also responsible for the formation of new concepts; the process of dissection is, indeed, exemplified many times by Frege when exemplifying how we can arrive from a proposition taken as a whole to an "unsaturated part", i.e. to a new concept (he was taking into discussion the possibility of many combinations created by his new approach upon the formal language of logic). For example, a sentence expressing a relation between two objects can be "dissected" in three ways, having one object replaced by a variable, the other object or both objects replaced by a variable. Dummett's verdict is that

> Dissection is therefore justly described as a process of concept-formation: it reveals something new, one pattern among many discernible in the proposition and shared by it with others, but not, in general, intrinsic to a grasp of its content[8].

It is apparent from these considerations that the process of "dissection" is the one revealing the pattern and the pattern is the one sustaining the process of inference. Because the patterns emerge like something new, the process yields

[6] *Ibidem*, p. 38.
[7] *Ibidem*, p. 42.
[8] *Ibidem*, p. 41.

information; but because they are already present in the sentence to be dissected, the problem of validity is solved:

> Such was Frege's solution to the problem of the utility of deductive reasoning. (...) All conceptual thought involves the imposition of form upon an amorphous reality on Frege's account, deductive reasoning requires the further imposition of form upon thoughts. It is surely that conception that can alone explain how such reasoning can be at the same time fruitful and cogent in virtue solely of the contents of the thoughts involved.[9]

2. DUMMETT ABOUT HUME'S PRINCIPLE

However, Dummett seems to deny that this procedure, of pattern recognition, is the one used by Frege also when describing his method of "carving up content". According to Frege, "carving up content" is the procedure used in the famous identity between "line a is parallel with line b" and "the direction of line a is identical with the direction of line b":

(a//b) = (Dir of a = Dir of b)

Frege claims that the same process takes place in his first definition of number (i.e. what was later called Hume's Principle):

(F equinumerical with G) = (Number of F = Number of G).

Consequently, the procedure of pattern recognition would not be an appropriate explanation for the informativeness of the above identities. Moreover, the acceptance of these identities together with Frege's affirmation that they share the same content (while their informativeness is entirely accepted), it would be "an aberration incompatible with his [Frege's] other doctrines"[10]. In other words, Dummett agrees that in Fregean framework there is a way of providing a solution for the paradox of analysis (if not one that would perfectly fit into the framework, at least one going in the same direction, or along the same lines), but this solution would not work for the first definition of number presented above because it would bring contradictions inside the framework.

It must be said that Dummett works with two identities where Frege presents only one. Namely, he 'splits' the so called Hume's Principle (or "the first definition") into two identities: the definition of the cardinality operator and the definition of the cardinal equivalence. He calls 'the definition of the cardinality operator' the equivalence between 'The number of Fs = the number of Gs' and 'there are just as many Fs as Gs'; the definition of cardinal equivalence is the other half of Frege's definition: "'There are just as many Fs as Gs' as meaning 'There is a one-one relation R whose domain consists of the Fs and whose range consists of the Gs'"[11]. The reasons for denying a solution to the paradox in the case of the first Fregean definition are specific for each of the two above definitions.

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[9] *Ibidem*, p. 42.

[10] *Ibidem*, p. 176.

[11] Michael Dummett, "Frege and the Paradox of Analysis", p. 41.

For the case of the cardinality operator, the dilemma is presented by Dummett in the following way: either the newly obtained concept was already presupposed in the grasping of the first part of the identity ('direction' already present in parallelism, for example); or it was not presupposed, but then, in order to understand 'direction of a = direction of b' we should not need the concept of 'direction'(only in this way the concept could be said to remain "the same'). But, according to Dummett, we have to have first the concept of 'direction' in order to be able to understand 'direction of a = direction of b'. otherwise, it would mean that we are capable of grasping the whole without grasping its parts, which would contradict the principle of compositionality that Dummett attributes to Frege[12] (this is the "aberration incompatible with Frege's other doctrines"):

> It clashed, in particular, with the doctrine, constantly repeated by Frege during his middle period, that the sense of a part of a sentence is part of the thought expressed by the whole. This doctrine means nothing if it does not mean that a grasp of the thought depends on a grasp of that constituent sense.[13]

Dummett believes that there is no way out by taking the first horn of the dilemma, so that the only way out would be the second. One way would be to say that the *content* is the same while the *sense* of the two parts of the cardinality operator is different (allowing in this way for the integration of the new concepts into the picture). But Dummett rejects this way out:

> Frege at no time proposed a distinction between the sense of a sentence and its content that would allow us to hold that, while a grasp of the sense of a sentence (...) involved having the concept of a number, which a grasp of the corresponding sentence did not, still the content of the two sentences coincided.[14]

His verdict is that the process that yields new concepts in the case of the definition of number might be called "concept formation", but is different from the one he described as giving the solution for the paradox (i.e. pattern recognizing); moreover, this other process does not have any of the guarantees that the process of pattern recognizing had, because it depends upon "dangerous ontological assumptions". I will not discuss the details of the problem regarding the dangerous ontological assumptions (demanding a certain status of the domain of the objects). The conclusion is that Frege was wrong to maintain the requirement that the two members of the equivalence have the *same content*.

The Fregean classical solution for informative identity statements, famously presented in 'On *Sinn* and *Bedeutung*', is that they have the same reference (*Bedeutung*) and different senses. However, this yields a somehow strange result when applied to the case of identity between statements because it would seem to result in all equivalent statements being identical (because they have the same reference, i.e. the

[12] Against the thesis that Frege would subscribe to this principle see David Bell in his article "The Formation of Concepts and the Structure of Thoughts".

[13] Michael Dummett, *Frege: Philosophy of Mathematics*, p. 176.

[14] *Ibidem*, p. 173.

same truth-value). Therefore, the almost unanimous approach is to search for a relation weaker than synonymy, but stronger than mere equivalence.

Until now only the first half of Frege's first definition in Dummett's interpretation was discussed. The other half, separately treated in his article "Frege and the Paradox of the Analysis", is the one named 'the definition of the cardinal equivalence'. Dummett actually states that:

> It is the definition of cardinal equivalence that raises the paradox of analysis more acutely than that of the cardinality operator. The definition is of course in terms of one-one mapping: specifically, it explains:
> There are just as many Fs as Gs as meaning.
> There is a one-one relation R whose domain consists of the Fs and whose range consists of the Gs.[15]

The problem here is the difficulty of accommodating Frege's criterion of sense identity (as understood by Dummett: "anyone who grasps the senses of two expressions must be able to recognize that they have the same sense, if they do"[16]) with the informativeness of conceptual analysis. In other words, if we are capable of immediately recognizing, just from the expression "there are just as many Fs as Gs", that this is about a one-to-one relation of correlation, then we do not acquire anything new by the above definition. However, the principle given by Frege would require this immediate recognition. Dummett's conclusion is that Frege is wrong to maintain this immediacy because recognition of synonymy might require reflection and inferences:

> The principle that anyone who grasps the senses of two expressions must be able to recognize that they have the same sense, if they do, is enormously compelling: yet to maintain it appears to rule out the possibility of conceptual analysis, of which the definition of cardinal equivalence is surely a prime example. Certainly, we must allow that, when the senses of the two expressions are the same, it must be possible for anyone who understands both expressions to come to recognize that their senses are identical: for, if this were not so, how could any conceptual analysis be established as correct, or, for that matter, as incorrect? Frege was anxious to insist that a recognition of synonymy must be immediate; if a proof is needed, we have, not synonymy, but the weaker relation of analytic equivalence. Against this, it is necessary to maintain that reflection may be needed in order to recognize the synonymy of two expressions; if this blurs the boundary between synonymy and analytic equivalence, that is a price that must be paid.[17]

Dummett discusses the abilities of a child who could know very well what "just as many" means without knowing that this involves a one-to-one correlation. This conclusion resembles the pattern recognition theory: the solution is based on the unawareness of the subject of connections brought to light by the conceptual analysis.

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[15] Michael Dummett, "Frege and the Paradox of Analysis", p. 41.
[16] *Ibidem*, p. 51.
[17] *Ibidem*.

> But what one grasps in practice need not always come overtly to mind when one is asked how to explain an expression, or even whether a given explanation is correct. That is why conceptual analysis can bring to light connections that we have failed to notice between different components of the complex linguistic practices we have mastered, and even abilities that we have failed to observe that we possess.[18]

To summarize, Dummett believes that the process of conceptual analysis can be saved from the accusation of "unfruitfulness", but not in a way that would result in coherence for the Fregean philosophy of mathematics. On this view, Frege would be right in proposing a 'recognizing pattern' type of solution, but this would not work for the two principles incorporated in the first definition of number, for different reasons for each of them.

I believe it is obvious that Dummett offers here a solution based on minimum changes (i.e. of the type same sense-same reference). He was making clear that he takes the requirement of having the same *content* to mean that the statements involved in the identity have the same *sense*. This is why he manifests surprise at Frege's (alleged) assertion of *synonymy* between (a//b) and (direction of a = direction of b)[19] and tries to explain it.

Therefore, I believe it is correct to say that Dummett's solution is based on the view that the informative identities are of the type same sense-same reference. Their informativeness, according to Dummett, does not come from a difference of sense, but from "pattern recognition":

> Frege's solution involved precisely the idea of discerning a pattern within a thought, or, rather, in the terminology of the early period, a judgeable content, a pattern is shared with a certain range of other thoughts or contents.[20]

This solution will be critically discussed in the next section. The main criticism brought against it is that the informativeness (i.e. the new piece of information obtained) explained in this way has a psychological flavor incompatible with a Fregean point of view.

4. CRITICAL VIEW OF DUMMETT'S SOLUTION

To remind, the solution of pattern recognition was one based on the distinction between form and content. The pattern discerned in a text is the logical form "hidden" in that text or, according to Dummett the pattern can be the hidden connection between truths in the demonstration of a mathematical problem (in this case, recognizing the pattern would amount to finding the demonstration). The point of this solution is that we are many times *unaware* of the rules that we use, i.e. of the patterns present in our usage. Therefore, when we do discover the patterns, this is a

[18] *Ibidem*, p. 52.
[19] See page 171 in *Frege: Philosophy of Mathematics*.
[20] *Ibidem*, p. 38.

genuine step forward for our knowledge. But, because the pattern was already there, present before our extraction, the validity of the process is secured.

I believe that, even if we accept that "pattern recognition" is a correct description of a process taking place when we perform an analysis, it cannot be a solution along Fregean lines because it comes in direct contradiction with one central Fregean tenet; namely, with the first principle of *Grundlagen*, stating that we have to always distinguish between logical and psychological, between subjective and objective. Let me elaborate on this point.

When Dummett gives his solution of "pattern recognition" he seems to forget an important distinction. This distinction started to receive explicit formulations with Leibniz, had several emphatic formulations in Frege and was brought forward as a coined expression by the members of the Vienna Circle. It is the distinction between the 'context of discovery' and the 'context of justification'. The well-known effect of this distinction is such that it draws a clear boundary between the actual process that leads to obtaining a certain piece of knowledge (the process being, inevitably a psychological one, belonging to certain subject) and the justification that can be offered for the truth of that same piece of knowledge (which justification is not supposed to be subjective, i.e. valid for only one epistemic subject, but intersubjective). As Frege famously maintained, for any simple arithmetical operation (let us take the famous 5 + 7 = 12) or any theorem (e.g. Pythagoras) we may describe the actual process of arriving at the result, or, alternatively we may give the justification for the result (i.e. the demonstration). Applied to the examples given, we may describe how we count 5 and 7 pebbles or how children learn to count; this will be always different from the demonstration of the fact that 5 + 7 = 12. These are simply two different perspectives upon the problem, answers to different questions. A clear formulation of the Fregean position can be found in *Foundations of Arithmetic*:

> It is not impossible that the laws of number are of this type. This would make them analytic judgements, despite the fact that they would not normally be discovered by thought alone; for we are here concerned not with the way in which they are discovered but with the type of ground on which their proof rests; or, in Leibniz's words: "the question here is not one of the history of our discoveries, which is different in different men, but of the connection and natural order of truths, which is always the same."[21]

Of course, there are cases when exactly the *origin* of a certain truth or class of truths is provided as a justification for, say, their a priori character. But this is exactly the situation that the distinction comes to clarify. How is this applying to Dummett's case? The process of pattern recognition, as presented by Dummett, is the *description* of the actual process that happens when a subject is (supposedly) acquiring a new piece of knowledge. To remind, the novelty was obtained from our *becoming aware* of the pattern present, but unnoticed:

[21] Gottlob Frege, *Foundations of Arithmetics*, p. 23.

> The characteristic of a pattern is that it is there to be discerned, but that, to apprehend that in which it is a pattern, we do not need to discern the pattern; it is essential to our discerning of the pattern that we recognize that that in which we have discerned it remains unaltered. One can hear a poem without identifying the metre or the rhyme scheme; someone unfamiliar with the Fibonacci sequence may fail to detect the principle determining the terms. [22].

The description as such might a correct one: most probably, most of the times we are *unaware* of the rules we use in making inferences; moreover, it is correct that extracting a pattern can make us aware of those rules. But *becoming aware* is essentially a psychological process which has nothing to do with the information conveyed by the sentence *per se* (the sentence could be false or true regardless of our *awareness* of this fact). Dummett himself states very clearly that all information was already there from the beginning and the only modification is that now is being discerned: "Since the relevant patterns need to be discerned, such reasoning is fruitful; but, since they are there to be discerned, its validity is not called in question."[23] Therefore, from the two characteristics required to be accommodated (informativeness and certainty), certainty is the one accounted for in a non-psychological way, indeed. This is because the validity of the equality is based on the pattern being present on both sides of the equality. But informativeness is clearly explained by appeal to psychological processes described before (becoming aware, finding something hidden, discerning). In other words, the novelty is not to be found *in the information per se* (clearly the information was already *there*, i.e. nothing *new* appears), but only in the subject performing the task of recognition of patterns, i.e. in the process of learning. But the process of learning, say, mathematics, is obviously different from its demonstrations, even though sometimes it is the demonstration that we learn. It appears that, for Dummett, the novelty does not reside in the information itself (i.e. there is no *new* information) but only in the subject processing the information[24] (i.e. there is only a *new* acquisition by someone of the *old* information). It is easy to see that it is not the quantity of information that has increased, but the number of the subjects possessing that particular information that has increased. Dummett's observation regarding the 'cardinal equivalence', namely "there are just as many Fs as Gs = there is a one-one relation R whose domain consists of the Fs and whose range consists of the Gs", is equally and more obvious connected with the psychological observation of the process of learning. His tenet was that the synonymy between the two parts of the definition of the cardinal equivalence is not immediately noticeable and requires

[22] Michael Dummett, *Frege: Philosophy of Mathematics*, p. 37.

[23] *Ibidem*, p. 42.

[24] In defense of Dummett's solution it might be said that pattern recognition is supposed to account for demonstrations, too (Dummett's examples are presenting it in this way). In this case, indeed, the solution would be placed in the context of justification, not in the context of the genesis of our knowledge. If by recognizing patterns what we actually do is providing a demonstration, then this is the correct way of regarding the problem (i.e. we are, actually, providing a justification for the result). My objection in this case is that a demonstration cannot consist simply in recognizing one and the same pattern repeated all over again. Dummett's affirmation that pattern recognition is a necessary condition for carrying out proofs is very probable to be true. But I do not think it is also a sufficient condition.

reflection (it is given the example of a child who may well know one, but not the other). This is the ground on which analysis is declared fruitful: though the two parts of the definition have the same content, it is necessary a considerable intellectual *effort* in order to recognize that. It is the same type of solution adopted: the information already there requires effort from the subject in order to be discovered; novelty is just the discovery of something already there, but which is "hidden" due to a failure of human capacities:

> But what one grasps in practice need not always come overtly to mind when one is asked how to explain an expression, or even whether a given explanation is correct. That is why conceptual analysis can bring to light connections that we have failed to notice between different components of the complex linguistic practices we have mastered, and even abilities that we have failed to observe that we possess.[25]

Jamie Tappenden in his article "Extending Knowledge and 'Fruitful Concepts': Fregean Themes in the Foundations of Mathematics" has a similar position, a similar division but different terms. His distinction is made also between a psychological and an epistemic point of view when evaluating the fruitfulness of knowledge; the expressions used are "the psychological advantage view" and the "conceptual setting view":

> I will call this the "psychological advantage" view; that people who think like us have cognitive limits, and the value of mathematical reasoning for broader intellectual endeavors is to be explained solely in terms of how it allows us to cope with these limits. I will sketch here an opposing position – the "conceptual setting" view – that takes to be objective aspects of mathematical patterns of reasoning (many of them difficult to nail down precisely, of course) that are responsible for the value of good mathematics. What distinguishes good mathematics from mathematics that is logically acceptable but unilluminating is in part that the good mathematics isolates the "right" concepts, classifications and theoretical contexts, where this "rightness" is not to be understood just in terms of our psychological quirks and shortcomings[26].

He also brings very convincing arguments for the view that Dummett[27] has a position corresponding to the "psychological advantage view", which view is in direct opposition with the Fregean understanding of the problem:

> These remarks [Dummett's] echo Hahn's: the informativeness of deductive conclusions turns on the "difficulty of establishing them and the surprises that they yield". Frege's understanding of the problem is different: the issue he addresses, and the solution he provides, should not be understood in this psychological way. Rather, he takes there to

[25] Michael Dummett, *Frege: Philosophy of Mathematics,* p. 52.

[26] Jamie Tappenden, "Extending Knowledge and 'Fruitful Concepts': Fregean Themes in the Foundations of Mathematics", p. 447.

[27] Tappenden affirms at a certain point that: "The subtle discussion in Dummett (...) helps bring into relief how easy it is to fail to distinguish the two approaches (...)".

> be a fact about what the structure of potentially enlightening inferences are, which his logical system captures.[28]

Also about the psychological novelty, extension of knowledge and surprising conclusions:

> Frege's attitude to "extending knowledge" by "drawing new boundaries" is objective/non-psychological in this way. Straightforward psychological accounts would not yield the distinctions Frege draws between Boolean combinations and quantificational definitions. A complex Boolean combination of properties may be more difficult to process and support more surprising conclusions than a simple quantifier expression. Yet the second will support extensions of knowledge because inferences directly involving it "draw new boundary lines". What it is about a fruitful concept/definition/principle which yields "extensions of knowledge" *is only incidentally connected with our apprehension of thoughts as psychologically novel*. The Weierstrass theorem is a paradigmatic analytic judgement "extending knowledge" because of objective facts about the structure of its derivation, not because its proof is difficult or surprising or anything else turning on psychological quirks of thinkers like us. [29]

The point in bringing this criticism to Dummett's solution is, as stated in the beginning of this section, to clarify what counts for this research as 'new' and what does not. The above criticism makes plausible that Dummett's solution is not a solution that could have been endorsed by Frege or that could have been designed along Fregean lines. Leaving aside the considerations regarding psychologism and the Fregean opposition to such procedures, we might very well accept that there is something new about this whole process and the process of learning cannot be said, after all to be "uninformative" (though it is not "informative" either, in the sense required by the justification context).

However, a solution to the paradox of analysis in Fregean terms requires a different kind of *new*, something that I will call "radical novelty" – and this is the clarification I was aiming at. The question if the Fregean framework has or not the theoretical resources necessary in order to solve the paradox is an important question and a problem to be pursued. However, the point of the present research might be conceived independently from it. Namely, the point would be to find out if a solution to the paradox of analysis is possible even in the situation when the radical kind of novelty is required. For let us suppose that the above type of solution (i.e. giving the type of answer 'already present but hidden') is indeed a possible solution. The point of the research, then, could be reformulated in the following question: 'do we have a solution also for a stronger requirement?'. The stronger requirement would be, of course, to allow the status of 'novelty' only for "something that has not been there already", excluding the situations of "something that was there but unnoticed before" (i.e. the weaker sense). In other words, the question remains: 'do we have a solution

[28] *Ibidem,* pp. 446–447.
[29] *Ibidem,* p. 450.

if we require a non-relativistic, non-psychological and non-subjective type of novelty (i.e. a 'radical novelty')?

In conclusion, Dummett's solution to the paradox of analysis – given in terms of finding hidden logical patterns – might be an answer to the question "how can an identity statements provide a new piece of knowledge?"; but it cannot be said to be a *Fregean* answer to the paradox of analysis. And this is because, as I have argued, Dummett's solution relies on the presence of a feeling of surprise at finding a pattern that was already there. Frege would reject this kind of answer due to its obvious reliance on psychological features of the epistemic subject.

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