KANT'S ARGUMENT IN THE THIRD ANALOGY

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Abstract. Kant's argument in the Third Analogy is about the simultaneous coexistence of substances in space. There are different interpretations of Kant's objective in this section of the Analogies of Experience. I reconstruct Kant's argument to demonstrate that it is about the empirical determinacy of time. For the experience of time requires also the empirical objectivity of time across space by registering the "same time" in different spatial separated locations. The Third Analogy establishes that registering the same time demands a mutual interaction or communication between substances, and it is here interpreted as involving the instantaneous configuration of substances in space.

Keywords: Kant, Third Analogy, simultaneity, mutual interaction, space, time.

The Analogies of Experience in Kant's *Critique of Pure Reason* are about the relational determination of the temporal content of experience. This section is important for explicating of the role of the dynamical categories such as permanence, causation, and community in Kant's conception of the categorical determination of experience. Most commentators on the Analogies have focused on Kant's Second Analogy, which is about successive causation, but less attention has been given to the Third Analogy. Unlike the Second Analogy, in which a cause precedes its effect in time, the Third Analogy is concerned with the relation of community or reciprocity between substances. For it pertains to a temporal relation other than successiveness, namely simultaneous coexistence between interactive substances. Most interpreters of Kant's argument in this section have argued that this Analogy does not add anything of importance to Kant's overall purpose of establishing the determinative role of dynamical categories with experience. The reason is attributed to situating this Analogy with respect to Kant's intent of 'refuting' Hume's argument against the necessity of causal relations. For this reason, the Third Analogy appears unable to contribute to this refutation of Humean position.

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This interpretation ignores a fundamental aspect of Kant's theory of time determination, and that is the empirical determinacy of time. For Kant, the pure manifold of intuition of time demands a relational determination by the dynamic categories, and one important aspect of time is the experience of a "universal now" in which it exhibits the universal agreement of all clocks on registering the same instant everywhere. Along with successiveness, simultaneity relation is also a constitutive component in our experience of time. For realizing the objectivity of this experience, Kant argues it requires a dynamic and relational category of the understanding to make the experience of simultaneity or coexistence of different substances possible.

THE ARGUMENT OF THE THIRD ANALOGY

In the Third Analogy of Experience, Kant turns to the conditions under which the simultaneous coexistence of substances can be experienced. Continuing from the preceding Analogies of Experience – the First and the Second – Kant aims to provide a procedure through which the determination of time is possible. Therefore, similar to the Second Analogy, time is determined through the deployment of a relational category. In the Third Analogy, the category of community (*Gemeinschaft*) enables the cognition of the simultaneous coexistence of substances in space. Kant states the principle of the Third Analogy in the following way in the first edition:

> All substances, insofar as they are simultaneous, stand in thoroughgoing community (i.e., interaction with one another). (A211)[1]

But in the second edition, it is stated in this way:

> All substances, insofar as they can be perceived in space as simultaneous, are in thoroughgoing interaction. (B256)

The two formulations are distinct with regard to the addition of space in the second edition's formulation of the principle. In the second formulation, Kant includes space in the principle of simultaneity, demonstrating how the structure of space relates to the content of the principle of simultaneity. But as will become clear, Kant also employs the structure of time to illustrate the simultaneity-relation, which involves a causal determination in the form of interactive causality or thoroughgoing interaction (*durchgängige Wechselwirkung*). Thus, this Analogy invokes not only the structure of space, but also temporal content that is being determined alongside it. For this reason, the inclusion of space in the second

[1] For all the references to Kant's *Critique of Pure Reason*, I cite the translation of Paul Guyer and Allen Wood, Cambridge, Cambridge University Press, 1999.

edition neither alters nor adds to the principle of simultaneous coexistence as formulated in the first edition.

In the proof of the Principle of the Third Analogy, Kant begins similarly to the Second Analogy by directing the reader to the perception of objects that are considered successive or simultaneous. In the case of simultaneous coexistence, the perception of one object can reciprocally follow the perception of another object, and it is possible to direct one's perception from one object A to another B, or conversely, from B to A:

> I can direct my perception first to the moon and subsequently to the earth, or, conversely, first to the earth and then subsequently to the moon.' (B257)

In this case, both objects, as given in the temporal content of perception, are considered simultaneous. In a similar fashion to the Second Analogy, where the successiveness of perception is immediately given, Kant argues that such perceptual successiveness does not provide the justification for inferring objective succession rather a category of causality must be employed in order to justify the objective temporal succession of objects. In the case of simultaneous coexistence, the possibility to direct one's perception from one object to another implies that perceptual experience can provide awareness of the simultaneous coexistence of objects in space through the reciprocity included in the perceptual experience. But this awareness does not justify objective simultaneity or provide a necessary connection within experience of the objective simultaneous presence of objects. Empirical imagination in perception cannot provide sufficient ground to establish the coexistence of the objects of perception. Therefore, a conceptual rule is needed to make the necessary connections or establish the mode of a temporal connection within experience objectively. It must objectively reproduce this temporal relation in experience. A categorical synthesis of understanding does provide the justification needed to infer the objective simultaneity between objects. Since it is impossible to perceive absolute time and know the temporal position of each object in time except through their relational determination, then a relational category serves as the objective ground for attaining simultaneity:

> Consequently, a concept of the understanding of the reciprocal sequence of the determinations of these things simultaneously existing externally to each other is required in order to say that the reciprocal sequence of perceptions is grounded in the object, and thereby to represent the simultaneity as objective. (B257)

But the content of this concept of the understanding needs further explication, and it is about the community of substances (*Gemeinschaft*), in which it involves a reciprocal determination or mutual interaction:

> …the relation of substances in which the one contains determinations the ground of which is contained in the other is the relation of influence, and, if the latter reciprocally contains the ground of the determination of the former, it is the relation of community or interactions. (B258)

The content of this relational category is about the mutual and reciprocal determination between substances, where one substance causally influences another substance, and the second substance also causally influences the first substance. This causal tie goes in both ways, so that each substance is cause and effect in relation to the other. Kant argues that the concept of simultaneous coexistence generally expresses the condition that the perception of simultaneous presupposes the category of mutual interaction to ground the temporal relations objectively.

The other formulations of the proof of the Third Analogy follow a similar pattern but with more emphasis on the temporal aspect of the argument. In the following paragraphs, Kant argues that perceptual experience contains an indifferent order in the case of perceiving the simultaneous presence of objects in space. However, such indifference is not enough to establish objective simultaneity. He includes the successive order to demonstrate that a reversal of other order of perception necessarily implies that earlier the objects of perception must belong to a past time, because it is impossible to connect these two moments of perceptual experience except as following each other successively (A211). In this scenario, it becomes impossible to recognize the simultaneity of these objects, demonstrating once again the insufficiency of the immediate order of perceptual experience to ground these temporal relations. The supposed indifference in the order of perception is intended to indicate that such perceptual order is associated with the category of reciprocal determination or mutual interaction. Although indifference is much more like the case of irreversibility in the Second Analogy, it follows necessarily from the application of this relational category to experience rather than being taken by itself as an objective ground for the simultaneous coexistence of objects.

In the following parts of the Third Analogy, Kant expands on his proof of objective simultaneity. He argues in more detail that the concept of coexistence requires the assumption of a causal connection between substances in space, and that the idea of a dynamical space, a non-empty space, implies the denial of isolated substances in space. This critique is directed at the Leibnizian position of causal isolation of substances. The idea is that coexistence necessarily entails a rejection of the causal isolation of substances, and the reason is that it becomes impossible or indeterminate whether such substances follow each other or simultaneous. The assumption of reciprocal determination or mutual influence involves a causal openness that allows for the objective perception of whether these states of substances are successive or simultaneous. In this part of his argument, Kant also emphasizes the role of empirical synthesis in relation to denying the causal isolation of substances (B259). The temporal aspect of an empirical synthesis or perception for Kant is brought to illustrate that the succeeding moments of perceptual synthesis ought to

be objectively ordered, and the ground for this perceptual order presupposes the interactive causal activity between substances in space.

Kant brings the temporal aspect of simultaneity-relation more clearly when he argues that the temporal position of a substance must be inferred via interactive causality rather than being posited by the substance's mere existence.[2]. He states this position in the Third Analogy in the following way:

> In addition to the mere existence there must therefore be something through which A determines the position of B in time, and conversely also something by which B does the same for A, since only under this condition can those substances be *empirically* represented as existing simultaneously. Now only that determines the position of another in time which is the cause of it or its determinations. Thus each substance (since it can be a consequence only with regard to its determinations) must simultaneously contain the causality of certain determinations in the other and the effects of the causality of the other, i.e., they must stand in dynamical community (immediately or mediately) if their simultaneity is to be cognized in any possible experience. (A212-13/B259)

The bidirectional link of the causal relation between the two substances determines their positions in time. In the Second Analogy, the successive and asymmetric line of causality is treated as what determines the successive order of time, or it is the categorical or conceptual synthesis that makes empirical determinacy of time possible. However, in the Third Analogy, and especially in the previous passage, Kant argues that the concept of mutual interaction necessarily implies the determining the position of a substance in time. In this sense, Kant is making the distinction of mutual interaction and successive causality clearly. He explicitly intends to establish the necessity of mutual interaction as the ground for the cognition of the simultaneous coexistence of substances. The implication here is that the category of mutual interaction or reciprocal influence reproduces a necessary and significant structure of pure time, namely, the simultaneous coexistence of different objects in time, and such a feature of pure (or mathematical) time requires a category different from the category of successive causality to be empirically realized. This poses the interpretative question of whether Kant is successful in making the transition from successive causality to mutual interaction and whether simultaneity relation does need such interactive causality to be cognized. The import of Kant's claim regarding the necessity of employing mutual interaction is to demonstrate that pure time is not yet empirically determined without the category of mutual interaction.

[2] This is also an argument that Kant had used in his early works, where he argued that mere existence, which contains the substance's inner ground of existence, cannot alone determine the substance's temporal position without being in an interactive and causal relation with the other substances in space. See Imm. Kant, *New Elucidation* 1:410, in Imm. Kant, *Theoretical Philosophy: 1755–1770*, translated and edited by David Walford and Ralf Meerbote, Cambridge, Cambridge University Press, 2007.

In the first edition formulation of the proof, Kant proceeds by stipulating that the experience of empirical synthesis of the temporal positions of substances must be determinate. He argues that mere perception cannot establish such determinacy because the reversible order of perception associated with simultaneous coexistence contains a temporal delay that does not guarantee the simultaneous coexistence of the perceived objects (A211/A212). Therefore, to obtain this determinacy of time, there must be a different ground for the determinate order of time other than perception. There, Kant argues whether such determinacy is achievable through the mere existence of substances and the causal isolation of each substance. He contends that such causal isolation is insufficient to bring out the temporal determinacy of substances (A212/B259). There must be something else besides their existence to make the empirical synthesis of time a determinate synthesis. This is possible through the interactive causality between substances, which includes the idea of reciprocal determinations. In the first edition's proof, Kant intends to include the position of those who want to establish the simultaneity of substances without interactive causality show the insufficiency of this position in regarding the determinacy of the empirical cognition of simultaneity. But in the second edition, Kant's proof does not invoke the idea of isolated substances; instead, he follows the same structure of argument in the Second Analogy, which invokes the impossibility of perceiving the absolute positions of things and the lack of perception as an objective ground of simultaneity (B257/B258).

One of the more elaborate interpretations of Kant's argument in the Third Analogy is presented by Eric Watkins in his different writings on Kant's conception of causality. In his first presentation of Kant's argument of mutual interaction, Watkins proposes an alternative way to understand the "model" of causal interaction that Kant advances. Before presenting Watkins' suggested model of mutual interaction in Kant, it is important to consider the model with which Watkins intends to contrast with his interpretation of the concept of mutual interaction.

In "Kant's Third Analogy of Experience"[3], he takes this model to consist of the following: A substance S at t_1 causes the state of another substance S′ at t_2, and S′ in t_1 in turn causes S to be in a state at t_2. This simple model of mutual interaction, according to Watkins, seems at first sight to be sufficient to establish the simultaneity of both substances at each successive t_1 and t_2 respectively. But he argues that there is a "fatal difficulty" with this model, which is that by employing the time intervals between t_1 and t_2, it "smuggles in coexistence illegitimately." This represents a defect in this model of mutual interaction, and it also generates difficulty if we replace t_1 and t_2 with "before" and "after". In this scenario, the first

[3] I rely on Watkins' first presentation of his interpretation as his later writings (especially Eric Watkins, *Kant and the Metaphysics of Causality*, Cambridge, Cambridge University Press, 2005) do not show much change in his reconstruction of Kant's argument: Eric Watkins, "Kant's Third Analogy of Experience", in *Kant Studien* 88 (4), 1997, pp. 406–441.

causal tie that connects S with S′ occurs *before* the state of S′ in t_2, and S′ causing S to be t_2 is also before the later state of S. He concludes that this model of mutual interaction does not guarantee the simultaneous coexistence of the two substances in t_1 and t_2, making the following objections:

> 1. It has not been shown that the later state of substance S′ determined by the first causal tie is simultaneous with the later state of substance S determined by the second causal tie, but rather only that each one occurs after the other's initial state.
> 2. It has not been shown that the later states occur an equal temporal distance after the initial states. It is entirely possible on this model that the one later state occurs just a split second after its causally related initial state so one cannot infer the simultaneity of the later states from their occurring at an equal temporal distance after the initial states.
> 3. But even if one could determine equal temporal distances between both sets of initial and later states, one could not infer the simultaneity of the later states from this fact, since this inference requires the simultaneity of the initial states, which has also not been shown. Therefore, the initial states of substance S′, which is determined to be prior to the later state of substance S by the second causal tie, is not necessarily simultaneous with the initial state of substance S determined to be prior to the state of substance S′ by the first causal tie. This is because both states are determined as occurring only at some indeterminate time prior to the later states of substance S and substance S′.[4]

For Watkins, this model of mutual interaction is defective because it assumes the specification of the temporal position of the initial states of interaction between the pair of substances and thus cannot necessarily ensure the simultaneity of the later states in each substance. Equally important, the temporal distance between the initial states and the later states is not specified, since it is possible to have mutual interaction, yet the time-difference between the causal tie is not equal. This produces an unequal temporal delay between the initial and later states. Hence, Watkins argues that an alternative model must be proposed that avoids the pitfalls of this model of mutual interaction.

Watkins suggests an alternative model that contains a two-way causal relationship between the two substances instead of having two independent causal ties. This works to avoid certain problems associated with the previous model since it indicates the causal activity of both substances and thereby determines each other's position in time. Watkins argues that such a model does not presuppose simultaneity; for this purpose, he appeals to Kant's account of the communication

[4] E. Watkins, "Kant's Third Analogy of Experience", pp. 435–437.

of motion in *Metaphysical Foundations of Natural Science*. He argues that simultaneity is not assumed in this communication of motion and that the inference to coexistence is warranted accordingly. The other essential component of this proposed model is that the causal connections are not to be construed as a relation between two determinate events, that is, as a determinate event causing another event to occur. Watkins contends that through this model of mutual interaction, Kant is remedying a common misconception about the nature of causation. This conception of causation is articulated by David Hume, who assumes that causation is a relation between events. Rather, the model of causation that Kant wants to argue for is about the exercise of causal activity by a substance on another substance, and this relation comprises "…one of the causal ties that make up mutual interaction establishes only that substance S changes the state of substance S′ from *c* to *d*, not that any state of substance S is thereby changed in a determinate way." The implication of this model according to Watkins is that:

> …the activity of causing (the exercise of the force) of the 'cause' substance is not itself a determinate event.[5]

The consequence of Watkins' model is that the cause's temporal position is indeterminate, which resolves the problem of it being both prior to and after the occurrence, leading to a contradiction. In Watkins' reconstruction, this model does not fall into this problem because it construes the cause as indeterminate and therefore does not need to specify its temporal position to account for the coexistence of substances. It is possible, therefore, that the cause does not precede its effect.

Watkins' account successfully points out certain elements in Kant's Third Analogy that must be taken into consideration. But the model he suggests for mutual interaction is not sufficient with respect to the objective Kant sought to achieve in the argument of the Third Analogy. Let us set aside the debate over whether the causal connection should be between events or substances and focus on the idea of having a cause that is indeterminate with respect to its temporal position. The Third Analogy makes clear reference to the temporal position of both the cause and effect, indicating that a cause must be in a determinate temporal relation with respect to the effect. Replacing the time intervals of t_1 and t_2 with the temporal order of "before" and "after" does not resolve the problem. What is needed is an account that explains the causal connection between two substances that are both instantaneously connected in time with no temporal delay. This allows for inferring the time of one momentary state of a substance from another momentary state of a different substance.

[5] *Ibidem.*

A RECONSTRUCTION OF THE ARGUMENT

The argument of the Third Analogy presented in the first edition emphasizes that the causal isolation of substances does not provide determinate temporal positions of substances. The causal connection between substances, on the other hand, may provide such determinacy of time for the relations of successivity and simultaneity. Kant emphasizes the role of causal connections in producing simultaneous coexistence between substances, and such coexistence involves filling space with matter. By filling space with content, the empirical synthesis is not disrupted by temporal gap between its contents. In other words, the empirical synthesis of time becomes dependent on the causal and reciprocal determination between substances in space.

In the Third Analogy, Kant argues that the mere existence of a substance cannot determine its temporal position. It needs to be involved in a causal interaction with other substances in order for its temporal position to be determinable. This thesis of reciprocal interaction between substances gives empirical content to time-assignments because it postulates real physical interaction. Consequently, no causally isolated system can be assigned a temporal position relative to other systems. Thus, a line of causal relata connecting two systems or more is necessary to realize and exhibit the temporal unfolding of each system empirically. The time of one system can accordingly be normed with respect to other systems with which it is causally connected. An interaction, in this sense, must involve an exchange of a physical quantity (e.g., energy) to be empirically realizable. For this condition to be fulfilled in both directions, Kant envisages a situation where both substances act on each other in a spatial perspective within which both substances are considered active and where both objects are simultaneous. This spatial perspective indicates the relative temporal position of each system with respect to the other system but unified under one single time-coordinate.

In the First Analogy, Kant had discussed the idea of persistence with respect to the application of the relational categories to the time-manifold. The relevance of this Analogy here is that time relations presuppose a substratum through which the concept of persistence is ensured, and through which it is possible to ascribe simultaneity or successiveness to the manifold of appearances (A182/B226)[6]. This Analogy, therefore, expresses the idea of the conservation of *something* over time, for instance, energy or mass, and that quantitative notion of substance grounds the duration of temporal series in terms of persistence. Kant provided two formulations of this principle in the First Analogy, the first of which emphasizes the notion of a persistent object in appearances and that its changes are mere determinations. But

[6] The point of such claim is to establish that the temporal relations of succession and simultaneity are grounded on something which persists. For example, a temporal interval between two instants can be taken as indicating a change only on the presupposition of a unified temporal framework.

in the second formulation, Kant requires that the persistence of a substance be exhibited as a persistent *quantum* of substance, in which quantum can, therefore, neither increase nor diminish. It is only on the condition of the conservation of the quantum of substance that it is possible to ascribe temporal duration to it.

In space, the conservation of constant energy allows for the compensation of energy loss elsewhere, and such compensation assumes that a dynamical system is both spatially and temporally extended[7]. Concerning simultaneous temporal series, the conservation of energy is distributed across space, holding the parts of a dynamical system in one space, and mutual interaction preserves the same energy for the whole system over time. In other words, a single time-coordinate can be extended across space through mutual interaction as long as the parts of the system in mutual interaction are instantaneously connected. The relation of the First Analogy to the Third Analogy is demonstrated through the idea that the conservation of a quantum of substance is also assumed in the mutual interaction between two substances in space and that each substance involved in this reciprocal interaction is presupposed to preserve its temporal unity (or its magnitude) before and after the moment of instantaneous causal connection[8].

Returning to Kant's critique of causal isolation in the Third Analogy, we may recall that Kant draws our attention to the difficulty of characterizing the natural, or the unperturbed behaviour of an isolated system[9]. For he argues that the ascription of determinate temporal relations to a causally isolated system is incoherent. The alternative is to posit lawlike connections between different systems to produce background conditions that provide us with invariant conditions through the stability and balance of forces. For Kant, such a lawlike connection must be relational and not intrinsic to a single system – it necessarily involves a causal

[7] The spatial aspect of the First Analogy is pointed out by Melnick in his discussion of qualitative similarity as it can possibly be replaced by law-governed spatial connections between different states: Arthur Melnick, *Kant's Analogies of Experience*, The University of Chicago Press, 1973, pp. 72–77.

[8] Melnick suggests that in what he refers to as "temporizing procedure", a representation of here-now, or a spatial-temporal instant, includes a "crossings" between the histories (or the time-lines) of different substances, which assumes the "pasts" to be connected. He takes this points to the "crux" of Kant's argument for interaction: Arthur Melnick, *Space, Time and Thought in Kant*, Kluwer Academic Publishers, Dordrecht/Boston/London, 1989, p. 49.

[9] In the Third Analogy, on the critique of causal isolation, he states, "Now if you assume that in a manifold of substances as appearances each them would be completely isolated, i.e., none would affect any other nor receive a reciprocal influence from it, then I say that their simultaneity would not be the object of possible perception, and that the existence of one could not lead to the existence of the other by any path of empirical synthesis. For if you thought that were separated by a completely empty space, then the perception that proceeds from one to the other in time would certainly determine of the latter by means of a succeeding perception, but would not be able to distinguish whether that appearance objectively follows the former or is rather simultaneous with it." (A212) This impossibility of ascribing determinate temporal relations to an isolated substance can be applied to the state of this system, whether it is in a "natural" state or "accelerated".

connection with other dynamical systems; in other words, it contains the spatial-temporal coordination of the system[10]. The objective is not to deny that there is a default or normal behaviour, but rather to assert that such a default configuration of the system is connected causally with other dynamical systems in space and time. In this respect, the default configuration of the system is also a causal process since such configuration is about the stable conditions under which a system is moving inertially in space. In this case, construing the default configuration of the system does not require causal isolation. Instead, the stability of forces can play this function by creating conditions under which any perturbation can produce instantaneous effects on the other side. A reciprocal and mutual interaction is the causal relation needed for such stability and balance of forces between different dynamical systems that are moving over time[11].

In this respect, to think of a reciprocal interaction is to think of two spatially connected systems. They are connected by a line of causality. A privileged spatial perspective is where these two systems can be represented as active in relation to each other at the same time. Kant in the *Metaphysical Foundations of Natural Sciences* (thereafter *MFNS*), provided a concrete form of local motion in which the idea of reciprocal interaction is invoked, in which he added a differentia to the concept of matter as being movable *in space*. With this differentia, it is possible to concretely represent the concept of mutual interaction through the communication of motion between moving bodies. In the Fourth Proposition of the Mechanics chapters of the *MFNS*, Kant seeks to prove the law of equality of action and reaction, and in this section, he argues that such a law is a particular instantiation of the "metaphysical law of community"[12]. The proof proceeds to establish that the equality of action-reaction must be guided by an equilibrium rule, which divides one motion between the two interacting bodies. This equilibrium is reproduced in absolute space, where the center of mass represents the two moving bodies as equally active to each other.

In his proof in the *MFNS*, Kant deploys the Relativity Principle on at least two separate occasions, which we will discuss in reverse order, beginning with our present topic, dynamic interaction, and turning to the "Phoronomy" immediately

[10] Here Kant expresses an idea which was also articulated in Section §26 in B edition of the Transcendental Deduction (B162/163). The examples of freezing water and the perception of a house both include the claim that the structure of space and time in the sensible manifold imposes a structure on the matter of perception such that this matter of intuition is being elevated to be determinable by concepts of the understanding, and it is cognizable as being related in space and time through causal relations.

[11] In other words, such assumption can be thought of as the fragmentation or the pulling-apart of one system in different spatial coordinates. Thus, any instantaneous change in one part of the system affects the other parts, and instantaneous configuration of the system over space-time requires that its constant energy is preserved.

[12] Imm. Kant, *Metaphysical Foundations of Natural Science*, translated and edited by Michael Friedman, Cambridge, Cambridge University Press, 2004, 545 (p. 84).

afterwards. According to Kant, connecting two bodies is impossible when one is considered to be at absolute rest. To provide a precise characterization of local interaction, it must be reconstrued as involving a reciprocal and mutual interaction in which the line of causality is relativized between the two empirical points. Motion can be divided between the two bodies in an arbitrary number of ways, but the equilibrium principle can single out a unique one in which the motions are considered equal and opposite. Therefore, the communication of motion is construed as reciprocal, allowing for the conservation of physical quantity in this exchange[13].

The concept of reciprocal interaction can be viewed as an extension of the concept of collision, which corresponds to two trajectories, thus two time-lines (or "world-lines") that intersect at one point – simultaneity here is the geometrical identity of the intersection point of the two lines. Furthermore, associated with this intersection is a conservative exchange of "quantity of motion". Physical collision is the dynamic correlate to this intersection, where each world-line is associated with a causal process or chain[14]. The argument of the Third Analogy imposes that the motions of systems must be positioned relative to one single time-coordinate, in which each dynamical system, along with its time-line, must be represented as moving before and after the intersection, and it produces a symmetrical coordination of time. For this reason, the temporal equality of both systems is inferred by being connected with the other system, which is moving toward it, and in this case, both systems are positioned relative to each other.

As Kant argues repeatedly, isolated systems cannot be synchronized. However, a coordination of states between the two systems makes it possible to infer the time of whatever is connected with either system, since their frame is uniquely determined by the equilibrium principle. Therefore, if we take each body to represent a uniform process, each body in its motion goes through equal intervals

[13] Kant provides more details to this concept of interaction in the communication of motion: "All *active* relations of matters *in space*, and all changes of these relations, insofar as they may be *causes* of certain actions or effects, must always be represented as mutual; that is, because all changes of matter is motion, we cannot think any motion of a body in relation to another *absolutely at rest* that is thereby also to be set in motion. Rather, the latter must be represented as only *relatively at rest* with respect to the space that we relate it to, but as moved, together with space, in the opposite direction, with precisely the same quantity of motion in absolute space as the moved body there has towards it. For the change of relations (and thus the motion) between the two is completely mutual; as much as the one body approaches every part of the other, by so much does the other approach every part of the first." (*MFNS*, 545).

[14] Dowe endorses the theory of conserved quantity of causation, and in his account, a causal interaction is an intersection of worldlines (which expresses a causal process with uniform and conserved quantity) that involves an exchange of a conserved quantity. In this intersection, Kant also endorses the idea of mutual change of relations between two pairs of worldlines, but he relies on relativizing the causal line connecting the two empirical points. See Phil Dowe, *Physical Causation*, Cambridge, Cambridge University Press, 2000, p. 90.

of time in equal intervals of space before and after the impact, which connects these two world-lines[15]. But the coordination of the two moving bodies before and after the impact cannot be checked unless the causal connection is reciprocal, which means that the causal activity exerted by the cause is equal to the opposite causal activity produced by the effect on the cause. The two dynamical systems can then be considered in a relative spatial position to each other, and to apply the relativity principle in this way is to invest in the physical content of the relativity rule to produce a spatial perspective that subsumes another spatial point under it. In this sense, it is possible to "double" the relative space in two directions[16], where the two bodies involved in the interaction are regarded as the endpoints and the boundaries of the interaction[17]. However, in order to refer to the instantaneous configuration of the system as it is moving uniformly in space, it requires that such a causal process be determinable by the relative spatial positions of different systems connected with it, and that the concept of doubling relative space into two directions, implicit in the rule of the composition of motions in Phoronomy, provides us with the idea of a self-intersecting line in every inertial motion[18].

The idea of a self-intersecting line indicates that the time of sending and returning of a message through a causal process is identical in every inertial frame. Thus, it is possible to make the following claim: The elapsed time of the causal process is equal to the elapsed time of the moving body in its inertial frame, and since the time interval between before and after the sending of the message in a self-intersecting causal process is identical, then the instantaneous configuration and state of the body in its causal connection with other bodies is included in the causal relations. For it becomes possible to link the instantaneous configuration of one system with another and, therefore, any perturbation or wiggling of one side of the causal relation will immediately produce instantaneous effects on the other members of this causal community. Thus, coordination of time relies on this idea of a self-intersecting causal line in every frame, which makes it possible to include the instantaneous configuration of the system over time.

[15] This realizes what can be referred to as the 'linearity condition' which states that any motion which is taken as uniform motion in one inertial frame and that it represents a uniform straight line, it then must be viewed as such in any other inertial frame. This linearity condition is discussed with relation simultaneity in Brian Ellis, Peter Bowman, "Conventionality in Distant Simultaneity", in *Philosophy of Science* 34 (2), 1967, p. 123.

[16] Vuillemin provides an elaboration on the role of "doubling" space in Kant's parallelogram law for the composition of forces. See Jules Vuillemin, *Physique et Métaphysique Kantiennes*, Paris, Presses Universitaires de France, 1955, pp. 60–69.

[17] On this supposition, uniform and stable systems are to be taken as directed toward spatially located points to ensure temporal equality before and after the collision. David Hyder provides more elaboration on the idea of forces as spatially directed towards empirical points in his *The Determinate World: Kant and Helmholtz on the physical Meaning of Geometry*, Berlin, Walter de Gruyter, 2009, pp. 60–61.

[18] I borrow this term from Adolf Grünbaum, *Philosophical Problems of Space and Time*, D. Reidel Publishing, 1974, pp. 686–687.

As mentioned above, Kant first introduces his Principle of Relativity in the Phoronomy of *MFNS*, where he argues that since absolute space is not an object of experience, while sensible or relative space can be experienced, motion is observable only relative to a larger empirical space, or "frame", relative to which motion can be observed (*MFNS*, 488). Kant then uses this principle to derive principles for mathematically composing different motions by means of constructions. Since the Relativity Principle allows Kant to divide any motion and ascribe it either to the frame, or to the body, or to divide it among both within the frame, he concludes that it can be used to complement the composition. However, as we saw, this method does not involve or require an absolute rest frame, meaning that ambiguity remains that will only be removed in the Mechanics of the *MFNS*. Here, a reciprocal and mutual causal connection empirically realizes the equilibrium condition and leaves only one reference frame as an option.

Thus, as we have seen, the conservation of a physical quantity in a spatially extended system makes it possible to describe the temporal evolution of this system in relation to a single time- coordinate. As was pointed out earlier, unifying time means a single-time coordinate and therefore demands that we can describe the temporal evolution of spatially separated material points with one variable. Thus, it demands a criterion of whether they are "at the same time", and the conservation of energy or momentum of a spatially extended system the whole system can provide it.

In this relation between the instantaneous configuration of a dynamical system and the determination of the time that a causal process can take to carry information between spatially separated points establishes a link between determining distant simultaneity and causal processes, which for Kant, must be actions-at-a-distance. This relation was brought to light more clearly in Einstein's theory of Special Relativity which assumes limits to the speed of light, which in turn sets limits on all causal propagation. Thus, in Special Relativity the synchronization of spatially separated clocks depends on causal signals between clocks that take time[19]. In Newtonian mechanics, by contrast, there is no speed limit to a causal connection, which is why actions-at-distance are allowed. In order to define the concept "at the same time as", Einstein invokes the concept of a light signal that is to be sent from one spatial position to another, and time for this signal to arrive at the other position where it will be reflected back to the first position. For both Kant and Einstein, in other words, synchronization depends on causal connections, realized as an action-reaction relation. But for Einstein, even though signals must be sent back-and-forth in a two-way communication, there is a time lag between the "legs" of the interaction. According to some interpretations, the synchronization of

[19] Hans Reichenbach argues this synchronization involves causal chain, because it contains the sending of a signal from one distant place to another, and it requires beside the knowledge of the distance, knowing velocity of the signal. See Hans Reichenbach, *The Philosophy of Space and Time*, New York, Dover Publications, 1958, pp. 124–126.

different clocks requires that in the communication between the different readings of spatially separated clocks, the speed of the causal process must be known to correctly register the "objective time"[20].

This model of mutual interaction enables us to explain the synchronization at different spatial points without temporal delay. The concept of synchronization requires that events at two different places be "connected" which for Kant implies a conservation of energy through the intersection of two time-lines, and this is expressed through the equality of action - reaction. In this way, Kant has envisaged the possibility of realizing the unity of time across space through mutual interaction. For in this case, the causal line connecting two systems ought to be relativized and assumed to be directed toward empirical points. In this way, as the one line of causality is relativized, it is also symmetrically divided between two separate points. It would make it possible, therefore, to infer the properties of one space-time into the other, and thus the equality of the time - interval is preserved. The mutuality of relations between the two systems is then reproduced in absolute space, and in this way, it is possible to divide the causal line (connection) between two points symmetrically. This is referred to as the doubling space, which retains the idea of reciprocity and mutuality of action. Also, in the composition of motions, where the concept of doubling space involves the connection between two systems, demands each system must be considered as relatively moving in relation to each other. The concept of self-intersection allows for introducing action-at-distance since it implies that the time between sending and receiving information is identical in every frame, and since it is the case that a conserved system expresses an inertial and uniform behaviour only through mutual interaction with other systems, it is therefore possible to ascribe an equal time interval to the two systems that are mutually interacting. This is inferred via the property of doubling space in the reproduction of intersection in absolute space.

CONCLUSION

This reconstruction allows for addressing some worries regarding the import of Kant's argument in the Third Analogy and whether it achieves its objective by establishing simultaneous coexistence based on mutual interaction. By showing that mutual interaction is a physical correlate to a geometrical identity of an intersection between two bodies, Kant, whether in Phoronomy and Mechanics of

[20] Wesley Salmon argues that in Newtonian mechanics the measurement of one-way speed of light is possible, because in classical mechanics there is no limit to the speed of material particles, and it can accelerate to arbitrarily large velocities. However, considering that Special Relativity takes the speed of light to be a limiting velocity, it becomes impossible to accelerate any speed beyond this limiting velocity. See Wesley Salmon, "Clocks and Simultaneity in Special Relativity, Or, Which Twin Has the Time?", in *Motion and Time Space and Matter: Interrelations in the History of Philosophy and Science*, Ohio State University Press, 1976, pp. 508–545, pp. 518–519.

MFNS, has demonstrated that this intersection is constrained by the relativization of geometrical points, namely, these points are taken to be directed at other points. It indicates that the relativity rule, as a rule of symmetry, implies that the two spatial points are *a priori* directed towards each other in which the motion of one body is reinterpreted as a motion relative to the other body. As both representing the endpoints of the interaction, the identity of the intersection between the two bodies is the geometrical correlate of their instantaneous configuration at the same time[21].

This also shows that Watkins' worry with respect to establishing the equality of time - interval between the states of substances is unwarranted since it is demonstrated here that this temporal equality is ensured through the equal dissection or fragmentation of one causal line between two points, and that the mutuality of relation provides a constraint of the doubling of space in two opposing directions allows for inferring the properties of one space to the other, including uniformity and simultaneity, which justifies the idea a causal line as being self-intersecting. Watkins' mistake is to ignore Kant's insistence on the inclusion of the cause in describing the relation of mutual interaction.

Unlike the Second Analogy where the temporal position of the cause is taken to precede the effect, in the Third Analogy the instantaneous state of the cause is included in the form of interaction. For the simultaneity of two states in different spatially separated dynamical systems is established by assuming that each system has conserved its constant energy over time, namely, each system contains equal temporal interval of before-after structure. This explicates the relation of the Third Analogy to the asymmetric - time causality of the Second Analogy.

The Third Analogy contains an extension of the asymmetric - time structure of causality to the spatially extended systems (or substances), and it poses a constraint, that the event-causality in the Second Analogy must be referred back to a spatially extended systems, and since the principle of continuity imposes that the effects of a causal activity must be temporally successive, these effects of causal activity are to be understood in the Third Analogy as being relative to space, namely as being in space as well. This allows for redescribing the asymmetric causal influence in the Second Analogy as consisting of two spatially material points or (endpoints) of the causal relation (following the symmetrical nature of space), and that the spatial separation between the relata of causal relation in the Third Analogy asserts that both systems are temporally extended (containing a before-after temporal extension) through mutual interaction.

[21] Kant argues early in his philosophical development that these physical relations such as motion and rest are to be reinterpreted as relative quantities. For him (as in his Critical period) there is no absolute rest, and motion is relativized to a relation of mutuality where two spaces are connected. This ultimately relates to his rejection of the internal conception of force among Leibnizians and Newtonians, especially Leibnizians on their insistence of a metaphysical power (entelechy) which explains the dynamical laws of interaction.