HEURISTIC ASPECTS CONCERNING MATHEMATICS IN KANT'S WORK

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Abstract. This essay explores the intersection of Kant's engagement with mathematics and its heuristic dimensions in his (pre)critical theoretic philosophy. It examines Kant's approach to mathematical methods – *synthesis* and *analysis* – through four interconnected sections: highlighting the role of heuristics as problem-solving method, the first section connects heuristics to the ancient mathematical methods and their role in transcendental philosophy; the second section investigates Kant's pre-critical adoption of the *synthetic* method and the problem of its imitation in (transcendental) philosophy; the third section reveals in a new and systematic manner how Kant integrates the mathematical and Newtonian methods in his critical project; taking the previously mentioned sections as an experiment, the final section reevaluates the relationship between heuristics and methodology, emphasizing its implications for transcendental philosophy as well as for any perspectives on Kant's program.

Keywords: analytic, synthetic, heuristics, mathematics, Kant, experiment of pure reason.

BRIEF INTRODUCTION

As the title suggests, my essay brings together two aspects of Kant's thought: his well-known appreciation for mathematics and the lesser-known, so-called "heuristic" aspects of mathematics and of scientific experiment, mainly in his late critical philosophy (B *Critique*[1]). It is well-established that Kant's engagement with mathematics predates his *Inaugural Dissertation*[2]. What is of interest in this essay will be divided into four parts as follows: the first part will consider certain aspects regarding the relationship between heuristics and the two mathematical methods (synthetic and analytic) in Kant; the second part will address the issue of

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[1] I will use the standard notation of the A/B *Critique of Pure Reason* and the translation of P. Guyer and A. Wood (Cambridge, Cambridge University Press, 1998).

[2] See, for instance, Emily Carson, "Kant on the Method of Mathematics", in *Journal of the History of Philosophy* 37:4, oct., 1999, pp. 629–652.

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imitating the mathematical method in philosophy, focusing on Kant's engagement with the synthetic method up to his critical program; the third part will examine the heuristics of mathematical methods and Newton's in the critical project (the period between the two editions of the first *Critique*, along with the two intermediary works, the *Prolegomena*[3] and the *Metaphysical Foundations of Natural Science* (*MFNS*)[4]; the final part will revisit the results of the previous sections in a discussion on the relationship between heuristics and methodology, proposing a reevaluation of these concepts.

Specifically, in the first part, I will connect the term "heuristic," which I take here in its straightforward, classical sense as "a method for solving problems", with the ancient mathematical methods of *synthesis* and *analysis*, and the "fundamental problem of transcendental philosophy". I will argue that Kant's approach to mathematical methods is actually more profound than it has typically been understood (the influence also stems from mathematically guided scientific experiment[5]). In order tot do that, I will not refer here to the "Hintikka-Friedman tradition"[6], for example, but I will discuss Zeljko Loparic's standpoint as developed in his "Kant's Philosophical Method"[7]. He is the author in whom I found a view somewhat similar to my own regarding the influence of mathematics on (the problem of) transcendental philosophy, though only up to a certain point[8]. Therefore, I will emphasize the important resemblances and differences between our standpoints, as well as the implications on heuristics in Kant. In the second part, I will show that Kant did not fundamentally exclude, even during the pre-critical period, the imitation

[3] I will use the standard notation of the *Prolegomena* in the edition translated and edited by G. Hatfield (Cambridge, Cambridge University Press, 2004).

[4] I will use the notation from the *Metaphysical Foundations of Natural Science* in the edition edited and translated by Michael Friedman (New York: Cambridge University Press, 2004).

[5] See Marius Augustin Drăghici, "Kant on Metaphisics as Science", in *Revue roumaine de philosophie*, no. 2/2022, pp. 297–314.

[6] They sustain the logical role of mathematics' certainty in Kant's transcendental philosophy. See E. Carson, "Kant on the Method of Mathematics", pp. 629–630.

[7] Zeljko Loparic, "Kant's Philosophical Method" (I, II), in *Synthesis Philosophical*, I, 12 (2/1991), pp. 467–484; II, 14 (2/1992), pp. 361–382.

[8] Although I developed my ideas regarding the influence of the two mathematical methods on Kant's program in the early 2000s, I did not have access to Loparic's studies (which is why, until now, he has not been cited in any of my works). However, this fact may further validate the interpretive perspectives we share regarding the common position – up to a point. Evidently, I acknowledge Loparic's authorship of this path, which I have recently discovered to be relatively similar to my own, to some extent. Regarding the differences, they are significant, even fundamental – concerning not only the order and the reconfiguration, but also the theoretical framework in which these methods are embedded (for which Loparic and I provide different interpretations), as well as their roles in the structure and the theoretical cores of the *Critique of Pure Reason*. Moreover, in some of my previous works (for instance, "Kant on Metaphysics as Science"), I have reached unexpected and very important results regarding this reconfiguration. From my perspective, these results have a fundamental impact not only on the reception of the type of program proposed in the first *Critique* but also on the very validity of the B Deduction. However, I will return to this in future research. In this study, I will focus, in the final part, on the current concept of "heuristics".

of the synthetic (mathematical) method in philosophy. In the third part, I will argue that Kant adopted, in a certain way, both methods alongside Newton's mathematical method in the critical project (*Critique* B edition). Finally, in the last part, I will suggest that, regarding the heuristic aspects of mathematical methods for solving the problem of transcendental philosophy, the two interpretations raise questions not only about the concept of "heuristics" but also about the connection between heuristics and methodology.

I. THE TERM „HEURISTICS" AND ITS RELATION WITH THE SYNTHETIC AND ANALYTIC METHODS

As for the origin of the term "heuristics", the concept wasn't used in Ancient Greek, but we know that it comes from the Greek "heuriskein" (*εὑρίσκειν*), which means "to discover" or "to find." It is associated with the heuristic tradition since antiquity, exemplified by legends such as that of Archimedes and his exclamation "Eureka!" (I have found!). Although the first thinkers who indirectly discussed heuristics were philosophers like Plato[9] and Aristotle[10], who investigated how knowledge is attained through (in)direct methods such as maieutics, dialectics or induction, the term "heuristics" was not explicitly used in that period.

The connection between mathematics and the "fundamental problem of transcendental philosophy" has been highlighted in a way that aligns more closely with my own position, as I mentioned, by Zeljko Loparic[11] in his two texts on "Kant's Philosophical Method". In these articles, Loparic refers to George Polya's book *How to Solve It*[12]. It is known that this book is dedicated to the teaching methods of mathematics to students and pupils, or to the training of teachers and the presentation of mathematics to interested audiences. I will not refer to this, nor I will address here Polya's position regarding heuristics. I will solely mention that it is said that he was the first to explicitly and distinctly articulate this concept. The place where he does this is all the more suggestive as it is right at the beginning of

[9] See in standard notation Plato, *Meno* (sections 80a–86c); *Republic* (Books VI, sections 509d–511e, VII, sections 514a–521b).

[10] See in standard notation Aristotle, *Posterior Analytics* Book I, chapters 1–8 (71a1–76a6), and Book II, chapter 19 (99b15–100b15).

[11] In addition to the two texts I refer to, see also Zeljko Loparic's *A semântica transcendental de Kant*, published in its 3rd edition in 2005 by Unicamp/CLE (Campinas). The first edition was published in 2000, and the second in 2002 at the same publishing house in Campinas. References in parts III and IV of my paper will be made to the Portuguese edition, as the DeGruyter translation was not accessible to me at the time of writing this text (*Kant's Transcendental Semanthics*, 2024).

[12] Full title: *How to Solve It. A New Aspect of Mathematical Method*.; first edition in print from 1945 (copyright by Princeton University Press), second edition from 1957, in the first and second printing from 1971/1973. References in this paper will be made to the Expanded Princeton Science Library Edition, with a new foreword by John H. Conway, Princeton and Oxford, Princeton University Press, 2004.

the preface to the first edition of his work, signaling that Polya's entire undertaking is marked by this consideration. Below is the excerpt that suggests this important observation:

> The following pages are written somewhat concisely, but as simply as possible, and are based on a logn and serious study of methods of solution. This sort of study, called *heuristic* by some writers, is not in fashion nowadays but has a long past and, perhaps, some future.[13]

Regarding our inquery, I underlie here the fragment where Polya invokes heuristics in a small chapter of his book dedicated to Pappus, because *here* Polya seems to be the first to associate the ancient mathematical methods of *analysis* and *synthesis* and heuristics as a way of solving problems. In the seventh book of his *Collectiones*, Pappus reports about a branch of study which he calls *analyomenos*. We can translate it in English as "Treasury of Analysis", or as "Art of Solving Problems", or even as "Heuristic"; the last term seems to be preferable here. A good English translation of Pappus's report is easily accessible; what follows is Polya's free translation of the original text:

> The so-called *Heuristic* is, to put it shortly, a special body of doctrine for the use of those who, after having studied the ordinary *Elements*, are desirous of acquiring the ability *to solve mathematical problems* [my emphasis], and it is useful for this alone. It is the work of three men, Euclid, the author of the *Elements*, Apollonius of Perga, and Aristaeus the elder. It teaches the procedures of analysis and synthesis.[14]

Here, we observe an understanding of heuristics in a 'stronger' sense, as I would call it – a sense fundamentally rooted in solving *mathematical* problems in an *explicit* and *systematic* manner. This contrasts with what I would describe as the 'weaker' perspective of the term *heuristics* as it appears in contemporary analytic discussions about the modes of knowing[15]. From this standpoint, the 'weaker' sense delineates a clearer boundary between heuristics and methodology. However,

[13] G. Polya, *How to Solve It*, from the "Preface" of the first printing, p. vii.

[14] As we shall see, this is not just a slight adaptation (in the use of the term Heuristics) by Polya of Pappus's version of Euclid's definitions from (Th. L. Heath) *Euclid. The Thirteen Books of The Elements*, Cambridge, 1908 (second edition, translation, introduction and commentary by Sir Thomas L. Heath, Dover Publications, 1956, vol. 1, p. 138 – the 1956 edition retains the pagination of the original edition), in G. Polya, *How to Solve It*, p. 141.

[15] I will refer to this point in Part III in connection with some of Timothy Williamson's research. The distinction regarding heuristics here does not resemble Loparic's distinction between an engagement with the world through *heuristics* as a method of problem-solving (exemplified by the science of the ancient Greeks, particularly geometry and algebra) or through a 'wondered' (philosophical) *contemplation* aimed at seeking the truth about the representation of the world. Kant would align more closely with the former tradition. (see Z. Loparic, "On the Unavoidable Tasks of Pure Reason", in *Kant e-Prints*, Campinas, Série 2, v. 3, n. 2, pp. 193–196).

this seemingly minor modification of Pappus's statement is part of Polya's presentation, which, as he himself acknowledges, does not provide an exact quotation of the text as translated by Heath (we noted above Polya's clarification). I emphasize this aspect because, in Heath's authentic translation, we can identify a different meaning regarding both the sense and application of the method of analysis (*Treasury of Analysis*):

> The so-called *Treasury of Analysis* (*Λύσις Απορίεως*) is, in short, a specialized body of doctrine intended for those who, after completing the ordinary *Elements*, wish *to acquire the skill of solving problems involving the construction of lines.* ***It is useful for this purpose alone*** [my emphasis].[16]

Another observation is necessary here, namely that Polya's replacement of the phrase "skill of solving problems involving the construction of lines" with "ability to solve mathematical problems," while maintaining that this manner of solving problems "is useful for this purpose alone," significantly shifts the emphasis from a 'weaker' sense of heuristics toward a 'stronger' one (as it appears in Polya's adaptation). Furthermore, this 'weaker' sense, as I call it, bears certain similarities to Kuhn's "paradigm" as described in the second edition of *The Structure of Scientific Revolutions*. This resemblance lies in the fact that learning this method is intended for those seeking to acquire a skill for solving problems limited to constructions with lines – within a clearly defined context and only after studying Euclid's *Elements*[17]. These observations highlight an evident oscillation between two meanings of heuristics: one closer to methodology (the 'stronger' sense) and other (the 'weaker' sense) closer to the more common contemporary understanding of heuristics as an adjunctive, implicit manner of problem-solving (as addressed in the analytic philosophy). I believe this 'weaker' sense of heuristics has not been sufficiently analysed, and may shed new light on the relationship between heuristics and methodology. Moreover, in the current context of discussions about the epistemological status of AI, such a debate could reinvigorate and recalibrate certain epistemological issues raised by the historicist movement in the philosophy of science inaugurated by Thomas Kuhn and by Popper's perspective on the distinction between the context of discovery and the context of justification. However, this aspect will not be addressed here.

With respect ro Kant's thematization of the two methods, Loparic claims, in his analysis, that it is possible to trace the origins of Kant's philosophical method to the combined method of *analysis* and *synthesis* from ancient Greek geometry. I

[16] Thomas L. Heath, *Euclid. The Thirteen Books of The Elements*, p. 138.

[17] In the sense I refer to here, the solutions concern problems that can be delineated disciplinarily, as Kuhn does in terms of "disciplinary matrix" (in some fragments from the second edition of *The Structure of Scientific Revolutions*... and in a few other later texts). In this respect, see M.A. Drăghici, „Blaga's Epistemology and its 'Modest Relativism' In Philosophy of Science", in *Revue roumaine de philosophie*, 66, 2, 2023, pp. 333–361.

agree with his perspective. The combined method, which involves proper analysis or transformation and resolution (*analysis*), along with construction and proof (*synthesis*), allowed Greek geometers to address both theoretical and construction problems. I also concur with Loparic's assertion that Kant employed this same method, albeit adapted and transformed to some extent, during his critical period. This adaptation was aimed at solving the central problem of his transcendental philosophy, as articulated in the *Critique of Pure Reason* and the *Prolegomena*[18].

The ancient combined method of *analysis* and *synthesis* used by Greek geometers is briefly explained in an interpolation to Book XIII, Proposition 1, of Euclid's *Elements*[19]. However, the most complete description of it is found in Pappus's *Collection*. Here, I use the translation of Pappus' work by Thomas Little Heath[20], instead of the one by Hintikka and Remes that Loparic used. I will first provide the brief definitions from Euclid's *Elements*, followed by Pappus's presentation (in translation), with some paraphrasing from Polya's version.

Euclid's original definitions:

> **Analysis** is an assumption of that which is sought as if it were admitted «and the passage» through its consequences to something admitted (to be) true. **Synthesis** is an assumption of that which is admitted «and the passage» through its consequences to the finishing or attainment of what is sought.[21]

As other authors observed, these fragments are hardly intelligible; but we have here Pappus's fuller account:

Pappus's definitions:

> **Analysis** then takes that which is sought as if it were admitted and passes from it through its successive consequences to something which is admitted as the result of synthesis: for in analysis we assume that which is sought as if it were (already) done (*γεγονώς*), and we inquire what it is from which this results, and again what is the antecedent cause of the latter, and so on, until by so retracing our steps we come upon something already known or belonging to the class of first principles, and such a method we call analysis as being solution backwards (*ἀνάπαλιν λύσιν*), or *regressive* reasoning. But in **synthesis**, reversing the process, we take as already done that which was last arrived at in the analysis and, by arranging in their natural order as consequences what were before antecedents, and successively connecting them one with another, we arrive finally at the construction of what was sought (constructive solutions); and this we call synthesis.[22]

[18] Z. Loparic, "Kant's Philosophical Method" (I), p. 467.

[19] Th. L. Heath, *The Thirteen Books of The Elements*, p. 138.

[20] *Ibidem.*

[21] Euclid, *The Elements,* Book XIII, *apud* Th. L. Heath, *The Thirteen Books of The Elements*, p. 138.

[22] Pappus, in *ibidem*.

Polya's definitions:

> In **analysis,** we start from what is required, we take it for granted, and we draw consequences from it, and consequences from the consequences, till we reach a point that we can use as a starting point in synthesis. For in analysis we assume what is required to be done as already done (what is sought as already found, what we have to prove as true). We inquire from what antecedent the desired result could be derived; then we inquire again what could be the antecedent of that antecedent, and so on, until, passing from antecedent to antecedent, we come eventually upon something already known or admittedly true. This procedure we call analysis, or solution backward, or *regressive* reasoning. But in **synthesis**, reversing the process, we start from the point which we reached last of all in the analysis, from the thing already known or admittedly true. We derive from it what preceded it in the analysis, and go on making derivations until, retracing our steps, we finally succeed in arriving at what is required. This procedure we call synthesis, or constructive solution, or *progressive* reasoning.[23]

I have to say that Loparic interprets Pappus's text (translated by Hintikka and Remes) through the lens of Polya's "paraphrasing," which was shaped by Polya's research interests: to facilitate the teaching of mathematics by making it more attractive, Polya divided the *method* of analysis into *problems-to-prove* and *problems-to-find*. Loparic adopted *this* distinction from Polya instead of the original one, which is between *theoretical analysis* [progressive] and *problematic analysis* [regressive][24]. It is possible that this may have amplified what seemed confusing for Loparic in Kant's texts regarding the order and combination of the two methods. I attempt to clarify some aspects in this paper. Referring also to Polya's reconstruction of the fragments, I will highlight the confusion and ambiguity in some of the passages above, specifically those that retain exactly what remained from Pappus (in Heath's translation). These remarks pertain to both the characteristics and the order (relation of succession) of the two methods.

Thus, as seen in the fragment from Heath's translation, which I will call "original", the definition of the analytic method states that analysis results in what would have been considered the "result of synthesis". Here it seems that both *analysis* and *synthesis* lead to the same result – logically, this does not appear problematic. The issue arises in the description of *synthesis* (in the definition of the synthetic method), where it is stated that, "reversing the process, we take as already accomplished what was the final result of analysis" – that is, we take as the starting point what was the final result of analysis. But the final result of analysis is also the

[23] G. Polya, *ibidem*, p. 142.

[24] I emphasyze this because I diverge from Loparic at this point: his reformulation, whether subject to critique or not, leads him towards a semantics of Kant's *Critique*. In contrast, I argue that the two methods must be related to the "experiment of pure reason" from the B Preface (see part III of this essay).

result of synthesis (according to the "original" definition of analysis). So, how is it possible for the same (final) result of analysis to be, on the one hand, the result of synthesis, and on the other hand, for the same final result of analysis to be taken as already accomplished and constituting a starting point in synthesis? In other words, how can the result of synthesis, which is also the result of analysis, simultaneously be the starting point of synthesis?

The confusion in the text translated by Heath led some commentators, such as Polya, to interpret the phrase I emphasize here – "to something which is admitted *as the result of synthesis*" – as something akin to "to something which is admitted in *synthesis as a starting point*, i.e., *the result of analysis*". Polya's adaptation: "till we reach a point that we can use as a starting point in synthesis". I will return below to Polya's modifications.

The confusion can be clarified in three ways. The first approach appeals to Polya's paraphrase, which provides an answer (through "paraphrase") to the natural questions raised by the ambiguity in the text itself – Polya rightly claims that his adaptation lends logical and semantic coherence, which may even reflect the original meaning (though this approach accepts interventions in the text). The second approach involves reconstructing the definitions by invoking what I would call a "principle of equivalence", offering an interpretation that explains the meaning of the two definitions *without removing or adding words*. The third approach, which may be the most plausible, concerns the manner in which the definitions were compiled and arranged.

The first approach is beneficial because it provides a coherent and logical meaning to the two definitions and explains very well *what Polya intends* in his research. However, on the other hand, besides modifying the definition of analysis, it does not serve our purpose, as Kant certainly did not have access to such a reconstruction (as Polya's). The second approach lacks intrinsic clarity and requires an effort of abstraction and imagination, but it may align with my interpretation of how Kant integrated these two methods into his transcendental philosophy. Moreover, it preserves the original text (Heath's translation) intact. The third approach would work without altering the texts and would also align with the sequence (order) of methods in Kant. In any case, the three approaches cannot be regarded as equivalent. The first, which I would call "nominal", is provided within Polya's adapted context (he "paraphrases" the definitions in the context of constructing examples from mathematics, necessary for his goal: facilitating the teaching and learning of mathematics). The second, the "general" one, is broader and can be considered a "source" of inspiration for Kant in his combination of the two methods. Additionally, it can encompass the first approach without being equivalent to it. Finally, the third approach can explain the possibility of both previous approaches and their combination.

Thus, the explanation for the fact that the results of analysis and synthesis are "the same" and the beginning of synthesis coincides with its very result, as it appears in the original definitions, can be supported by the second approach as follows: we say that the result is the same not through identity but in different, yet equivalent terms in relation to the same "invariants". In other words, both the result of analysis and that of synthesis assume self-evident elements as structurally valid or fundamental constituents that are rationally generalizable. In this sense, the result of synthesis is the same as the starting point of the same synthesis, in terms of the equivalence of these "invariant" elements, structured (at the beginning of analysis) into what will subsequently be decomposed at the conclusion of synthesis, up to the "constructive solution". This explanatory approach, perhaps too abstract, is worth noting because, as we shall see, the "identity of results" of these methods (in the form of "categories and pure intuitions") represents a central idea in the mature program of Kantian transcendental philosophy. This idea, of the identity of the results (of the synthetic and analytic methods), is not found in the Hintikka-Remes versions nor in Polya's paraphrased version but only in Pappus's unrefined form as translated by Heath. Clearly, as I mentioned earlier, Kant would not have had access to the second approach; for this reason, I have emphasized this interpretation.

On the other hand, Polya's approach (the first version), which I will revisit now, as he himself specifies, is instructive even in the sense of observing (a hermeneutic effort) how Kant would have worked with this version, had he had access to it. What we do know for certain is his high regard for the synthetic method in mathematics and that he was most influenced by Newton's version[25], as well as by the "method of physicists and chemists" – aspect that I will discuss in section III. Thus, Polya mainly modifies the paragraph where it is stated that the result of analysis is the same as the result of synthesis and that the result of synthesis is also its starting point (which seems like nonsense). Polya simply says that the result of analysis is taken as the starting point for synthesis (which can be supported, since this element is explicitly stated in the definition of synthesis), but *without retaining that the result of synthesis is the same as that of analysis*.

Here are the two versions of the first part of the definition of the analytic method: „In **analysis,** we start from what is required, we take it for granted, and we draw consequences from it, and consequences from the consequences, till *we reach a point that we can use as **a starting point** in synthesis*." (**Polya's** version); „Analysis ***then*** takes that which is sought as if it were admitted and passes from it through its successive consequences *to something which is admitted as **the result** of synthesis*." (the **original** version). In the original text, analysis is preceded by synthesis, as indicated by the use of "*then*" – a term that is absent in Polya's version, precisely because he places analysis first! I believe this aspect is fundamental in our context, as it is also explicitly the case with Kant, where *synthesis* precedes *analysis*. On the other hand, the differences are not fundamental

[25] See also M.A. Drăghici, "Kant on Methaphisics as Science", in *Revue roumaine de philosophie*, no. 2/2022, pp. 297–314.

as for the definition of synthesis. I will only provide here one version to indicate that, in this case, the order seems reversed: *synthesis* follows analysis in succession and begins with the result of the latter. „But in **synthesis**, reversing the process, we take as already done that which was last arrived at in the analysis and, by arranging in their natural order as consequences what were before antecedents, and successively connecting them one with another, we arrive finally at the construction of what was sought (constructive solutions); and this we call synthesis".

Finally, the third approach I propose may help solve this puzzle, based on a reasonable assumption, namely that both definitions seem to have been *associated* and placed together in Pappus's texts, but *each one originating from a different corpus*. In this case, what is missing in Pappus's text, for example, the passage of analysis, was completed and placed *earlier* as another definition of synthesis, and vice versa: the fragment of synthesis associated with the fragment of analysis in Pappus's text would have been preceded by *another fragment* of the definition of analysis (such as the one adapted by Polya, which is coherent with the synthesis definition found in Pappus's text). Thus, it concerns missing fragments of the two definitions in Pappus, complementary to each definition, which are combined through *association* in Pappus's text – fragments that do not appear in either Pappus's version or in Polya's "paraphrase".

The complete picture would have looked as follows: there should have been two definitions for analysis and two for synthesis, in which, in one version, synthesis would come first, and in the second, analysis would come first. This would explain the paradox that the results of analysis and synthesis would coincide, given that the beginning of synthesis also coincides with its result: by *dissociating* the two fragments from Pappus, the fragment intended for analysis, which states that analysis reaches the same result as synthesis, is preceded by the missing fragment about synthesis, which could even be the brief version of Euclid's definition, where "synthesis is an assumption of that which is admitted 'and the passage' through its consequences to the finishing or attainment of what is sought". Therefore, we have two ways to reach "the same result": the first, where axioms, definitions, theorems, or propositions, which presuppose corollaries, consequences, algorithms for solving, etc.; the second starts with the problem itself, which is gradually brought closer to what the result of synthesis presupposes: an algorithm for solving compatible with the transformations of analysis.

In this way, the ambiguities pointed out by Loparic in his interpretation can be dispelled, according to which, most often, mathematicians either "omitted" the presentation of analysis before synthesis or presented synthesis before analysis. What I want to argue here is that both meanings can be accepted, with synthesis first or with analysis first, and in this way the "original" version stands, validating both Heath's incomplete translation and the need for logical and semantic coherence achieved through Polya's adaptation. Furthermore, in mathematics, the order is present in both directions, depending on the "problem", whether it is purely theoretical or refers to problems to be solved (as with Polya), or both versions[26].

II. METHODS IN PRECRITICAL KANT

With the third modality, I have shown above that the presentation of the definitions of the two methods depends on *the type of interrelation* and on the order of their succession (whether synthesis comes first or analysis comes first), and I have also emphasized the different way in which they appear in Heath's translation and Polya's paraphrase. Nonetheless, Polya acknowledges that he proceeded to clarify and specify the original text, noting that the initial text could, in fact, allow for a broader generalization and a validity of the methods that would extend beyond the typical framework of mathematical problems (which is particularly interesting for our subject).

What has been presented and discussed above will also be useful for introducing certain excerpts (mentioned by Loparic) from Kant's works during both the pre-critical and critical periods, concerning the two methods and how these methods may have influenced the mature project of the *Critique* B. Additionally, I have tried to highlight the source of Loparic's convictions regarding Kant's prioritization of the analytic method, followed by the synthetic method, despite many contrary Kantian assertions[27] – I reiterate that here I endorse a standpoint contrary to that of Loparic.

The way Kant was influenced by the two methods remains a subject of debate today[28]. The German philosopher did not take these methods "as such", as they appear to us today in Pappus, or as clarified, with a particular interest, by Polya[29]. This is the difficulty to which Loparic failed to respond and which probably led him to argue that Kant adopted the same order of methods as they appear in Polya's references to solving mathematical problems. Although in mathematics the methods do not always appear in the same order (Loparic also addresses this aspect and acknowledges that among earlier commentators, the analytic method is either merely mentioned or does not appear at all[30]), the two methods are presented in Pappus and Polya as *consecutive*, in a *sequence* that forms a chain, with Polya treating them as

[26] We shall see in section III that we can find in Kant all these versions.

[27] Z. Loparic, "Kant's Philosophical Method" (I), p. 472.

[28] See, for instance, Gabriele Gava, "Kant's Synthetic and Analytic Method in the *Critique of Pure Reason* and the Distinction between Philosophical and Mathematical Syntheses", in *European Journal of Philosophy*, vol. 23, issue 3, pp. 728–749. Either in this or other of his researches, I have not found common points or intersections with Loparic's position or my own (for instance, Gava considers the method of analysis as pertaining *exclusively* to exposition, without attributing to it deeper, mathematical connotations).

[29] In the explanation I provide immediately above, it is necessary to read the corresponding fragments from Polya, *How to Solve It*, pp. 142–148.

[30] Z. Loparic, "Kant's Philosophical Method" (I), p. 474.

two interconnected stages in problem-solving. In section III, we will see that, in Kant's work, these methods are initially separated due to reasons related to the nature of the Kantian object of inquiry (the object of Kantian analysis is the intellect and the pure reason, our competence in relating to objects, not a specific problem, as in mathematics) and the a priori transcendental theoretical framework within which the inquiry takes place. This is one of the reasons why Kant *must* initially use the *synthetic* method rather than the analytic method, as Loparic believes.

On the other hand, I agree in general with Loparic's statement that, though it seems impossible to place all of Kant's procedures under one general methodological scheme, there are good grounds to say that most tenets of both his speculative and practical philosophy were established by the method of *analysis* and *synthesis*, adapted from Greek geometry[31]. This way of understanding Kant's "royal road" to philosophy sheds new light on the structure of the problems he was solving, as well as on the natural order and dependence of his arguments. Moreover, understanding Kant's method of *analysis* and *synthesis* greatly aids in the study of other methods he employed. In the third part of this study I will resume some of the results from my previous research[32].

I also agree with Loparic that Kant, even during the period of the *Prize Essay*[33], was aware of the Newtonian method as the appropriate one for his critical project[34] which had yet to take shape. Before presenting my view on the famous fragment from Newton's *Opticks* in correspondence with what Kant argues in the B *Critique*, let us first analyze his position during the pre-critical period by examining excerpts from works such as *Physical Monadology*[35], *The Only Possible Argument*, *Prize Essay*, or *Lectures on Logic*[36].

What will be of interest in this part is not only identifying Kant's support for one method or the other or for a specific order but also addressing the problem of the "imitation" of the mathematical method in philosophy. This can further clarify the perspective(s) from which Kant evaluates the two methods – whether from traditional metaphysics or transcendental philosophy.

The issue of the "imitation" of the mathematical method in Kant's philosophy is a topic that has perhaps been somewhat overlooked in contemporary analyses.

[31] *Ibidem*, p. 467.

[32] M.A. Drăghici, "Kant on Metaphysics as Science".

[33] For ease of reading, I will use the title established in the exegesis, *Prize Essay* for the English translation („Inquiry concerning the Distinctness of the Principles of Natural Theology and Morality") of Kant's work *Abhandlung über die Evidenz in metaphysischen Wissenschaften* (1764). This work and others to which I will refer throughout this essay are parts of the well-known *The Cambridge Edition of the Works of Immanuel Kant, Theoretical Philosophy 1755–1770*, David Walford, Ralf Meerbote, (transl. and eds.), Cambridge, Cambridge University Press, 1992.

[34] Z. Loparic, "Kant's Philosophical Method" (I), p. 476.

[35] These works were first considered in this subject matter by E. Carson in her article.

[36] I will refer to some of Kant's works on logic translated and edited in *The Cambridge Edition of the Works of Immanuel Kant, Lectures on Logic*, Paul Guyer and Allen W. Wood (general editors), Cambridge, Cambridge University Press, 1992.

Regarding the present endeavor, I believe it encompasses certain aspects that could prove productive. For example, answering the question of whether and in what way can we speak of the imitation of the mathematical method in philosophy sheds light on how Kant adopts the two methods in his critical philosophy. Of course, there is general agreement that Kant was primarily concerned with achieving the highest possible level of certainty regarding his philosophical system. Moreover, based on this aspect, we can provide additional arguments about the order of succession of the two methods beyond Kant's own testimony. As I have previously stated, I will argue that the first method in the sequence is synthetic, followed by the analytic, and then both combined.

The framing of this issue can be summarized in two distinct positions Kant held in two different works, separated by significant spans of time: the first characterizes an early work (*Physical Monadology*, 1756), where metaphysics and mathematics appear closely aligned in terms of achieving a comparable level of certainty; the second is Kant's standpoint in the B *Critique*, where he seems to lean more heavily toward mathematics.

In the *Physical Monadology*, Kant particularly addresses the relationship between metaphysics and physics, mediated by geometry, later linking geometry back to metaphysics:

> Philosophers who are clear-headed and seriously engage in the investigations of nature unanimously agree, indeed, that punctilious care must be taken lest anything concocted with rashness or a certain arbitrariness of conjecture should insinuate itself into natural science, or lest anything be undertaken in it vainly, without the support of experience and the mediation of geometry.[37] [1: 475]

A reference should be made here to another work of Kant's, prior to *Physical Monadology*, where this mediation becomes clearer, namely, that in a certain sense, the behavior of the planets under the gravitational influence of the Sun is mediated by geometry. This reference is included in a note by the editors of the English translation[38] of Kant's *Physical Monadology*. Regarding the relationship between metaphysics and geometry, as Emily Carson emphasizes – the author who will accompany us through this part of the analysis — although Kant highly appreciated mathematics, he highlighted the differences between the two, while also valuing metaphysics and wondering how it could be unified with

[37] Imm. Kant, *Physical Monadology*, in *Theoretical Philosophy 1755–1770*, p. 51.

[38] "A good example of the mediation of geometry is provided in Kant's summary, found in the preface to the *Universal Natural History* (1755) of the Newtonian world picture, as it applies to the solar system (AK I:243-6) Kant characterises the sun's gravitational influence on the planets at one point (AK I:243) as in a manner 'established... by geometry.'" (*Physical Monadology*, *Theoretical Philosophy 1755–1770*, p. 422, n.).

geometry[39]. In this respect, we revisit a corresponding fragment from the Preface to *Physical Monadology*:

> Metaphysics, therefore, which many say may be properly absent from physics, is, in fact, its only support; it alone provides illumination. For bodies consist of parts; it is certainly of no little importance that it be clearly established of which parts, and in what way they are combined together, and whether they fill space merely by the co-presence of their primitive parts or by the reciprocal conflict of their forces. But how, in this business, can metaphysics be married to geometry, when it seems easier to mate griffins with horses than to unite transcendental philosophy with geometry? For the former peremptorily denies that space is infinitely divisible, while the latter, with its usual certainty, asserts that it is infinitely divisible. Geometry contends that empty space is necessary for free motion, while metaphysics hisses the idea off the stage. Geometry holds universal attraction or gravitation to be hardly explicable by mechanical causes but shows that it derives from the forces which are inherent in bodies at rest and which act at a distance, whereas metaphysics dismisses the notion as an empty delusion of the imagination. [1: 475]

Moreover, as Carson also observes, Kant indeed seems to agree, at least here, with both mathematics and metaphysics regarding the issue of the infinite divisibility of space in relation to the status of the monad as composed of simple, indivisible substance. Kant's explanation involves a process of reconceptualization, but we will only briefly mention it here[40]. Here is his conclusion:

> But it is clearly apparent from what has been demonstrated above that it is neither the case that the geometer is mistaken nor that the opinion to be found among metaphysicians deviates from the truth. It hence follows that the opinion which divides them both, namely, that an element, which is absolutely simple in respect of its substance, cannot fill a space without losing its simplicity, must be false. [I: 480].

Although, in B *Critique* Kant states that:

> (T)he pursuit of the mathematical method in this sort of cognition cannot offer the least advantage, unless it is that of revealing its own nakedness all the more distinctly, and revealing that mathematics and philosophy are two entirely different things, although they offer each other their hand in natural

[39] E. Carson, "Kant on the Method of Mathematics", p. 630.

[40] In short, Kant demonstrates that the infinite divisibility of space (as asserted by geometers) does not contradict the simplicity of monads (as asserted by metaphysicians), since spatial relations do not imply a plurality of substances but rather reflect the manner in which substances interact with one another.

> science, thus that the procedure of the one can never be imitated by that of the other. [A 726/B 754]

However, I believe that the issue of imitating the mathematical method must be understood in a much more nuanced way. Therefore, I propose that we consider the following: Kant did not accept the use of the mathematical method (the synthetic method) in philosophy and metaphysics *as such*; he criticized the erroneous use of this method in metaphysics and maintained that the appropriate method for philosophy is the analytic one. On the other hand, he admitted the "application" of the synthetic method in metaphysics, but under certain conditions – he allowed the use in philosophy of results and knowledge derived from mathematics – for instance, concepts concerning quantities or references to certain results from physics. He also acknowledged that, although "it may take a long time for the synthetic method to be applied in philosophy", this possibility is not excluded. In the discussion that follows, I will attempt to clarify this issue so that we can coherently understand Kant's standpoints here.

What I intend to argue is that, in fact, Kant did not *fundamentally* exclude the presence of the synthetic method in philosophy, not even during the 1750^{s}. In this respect, I will refer to some fragments where this is fairly evident, as well as, at the end of the analysis, to the context in which Kant refers to Newton's method. These elements highlight how Kant adopted and refined the two methods across the two editions of the *Critique* (my focus in part III). Equally significant is the point that Kant not only embraces the lessons of physical science for metaphysics, but also acknowledges the reverse process, where metaphysical insights can inform and guide scientific inquiry.

Thus, I will consider these points to be entirely correct for, as stated in the opening of the "preliminary considerations" to *Physical Monadology*, what is borrowed from science through mathematics is later converted into dominant theses equivalent to the critical project. Kant asserts at the beginning of this work that "lucid philosophers" who seriously engage in investigating nature unanimously agree that nothing hastily fabricated should be allowed to infiltrate the science of nature in order to avoid any endeavor unsupported by experience and unmediated by geometry. Furthermore, Kant remarks that "surely nothing can be thought to be more useful or beneficial to philosophy than this advice" (from the viewpoint of the young Kant, we find here, *in nuce*, the project for reforming metaphysics, as outlined later in the *Critique of Pure Reason*). On the other hand, in *Physical Monadology*, metaphysics is valued at a level comparable to that previously attributed to geometry and experimental science. Kant states that although mathematized experience allows natural science to progress unproblematically in understanding through "the exposition of the laws of nature" (mediated by geometry), it is an error – especially from the perspective of the "scientific-dogmatic" thinker – to believe that one can also present "the origin and causes of these laws". Kant emphasizes that this misconception can only be corrected by the testimony of metaphysics. He observes that “those who only hunt out the phenomena of nature are always that far removed from the deeper understanding of the first causes”. They will never come to know the true nature of bodies, just as those who believe that “by climbing higher and higher up the pinnacles of a mountain they will at last be able to reach out and touch the heavens with their hands”[41] will fail in their endeavor. Natural philosophers, or scientists, must learn from the failures of metaphysics – or from the “awakened” metaphysician of later times – that the claims to knowledge cannot be extended beyond the boundaries of experience. This scientist appears to resemble the traditional (dogmatic) metaphysician, who seeks to know the first causes through experiment and mathematics, an effort just as futile as that of the dogmatic metaphysician who incorrectly borrows mathematical tools to support speculative, or experimentally uncontrollable assumptions[42].

I dare say that the above reflects a youthful thought of Kant’s, one less emphasized in exegesis, but which I consider necessary to highlight – not only to correctly evaluate the dichotomous relationship between philosophy (traditional metaphysics) and mathematics, often reiterated by Kant later on, but also regarding the “mutual borrowings” between the two disciplines. In this latter sense, the opening sentence of the fragment cited above must be read differently, as it might otherwise seem harder to explain: “Metaphysics, therefore, which many say may be properly absent from physics, is, in fact, its only support; it alone provides illumination.” The meaning of “illumination” coming from metaphysics, I believe, is this: insofar as the natural philosopher is dogmatic and believes he can know the first principles through experience and mathematics, he will err. Here, the “lessons” of the metaphysicians – who, if they investigate without limiting themselves to experience, will endlessly stray in the search for the truth of these first principles – provide illumination in the sense that the natural philosopher will realize that the only way to remain within the realm of certainty is to embrace experimentation and its data, which are, after all, their very objects and tools of work. The reverse is also true: metaphysics needs the data provided by mathematics and physics to avoid contradicting them and to take them into account[43], ensuring that it stays on a secure path. These elements are retained,

[41] Imm. Kant, *Physical Monadology*, in *Theoretical Philosophy 1755–1770*, p. 51, [1: 475].

[42] We find almost the same idea in B *Critique*: “Mathematics is thoroughly grounded on definitions, axioms, and demonstrations. I will content myself with showing that none of these elements, in the sense in which the mathematician takes them, can be achieved or imitated by philosophy and that by means of his method the mathematician can build nothing in philosophy except houses of cards, while by means of his method the philosopher can produce nothing in mathematics but idle chatter, while philosophy consists precisely in knowing its bounds, and even the mathematician, if his talent is not already bounded by nature and limited to his specialty, can neither reject its warnings nor disregard them.” [A 727/B 755].

[43] E. Carson (“Kant on the Method of Mathematics”, p. 632) summarizes this aspect very well; far from turning mathematics into a body of “subtle fictions” when it conflicts with the propositions of metaphysics, metaphysicians should view mathematics as a source of fundamental data: “Mathematics is thus the paradigm of certainty.”

in essence, even in the *Critique of Pure Reason*, specifically in B *Critique* (A 727/B 755; see note 42).

With respect to the differences that seem irreconcilable between metaphysics and geometry, as seen in the above fragment concerning the differing claims about the empty nature of space or the concept of universal attraction explainable through mechanical causes, Kant shows that in this formula, metaphysics and geometry cannot be unified within the traditional framework that was still dominating.

However, these are the extremes of the two standpoints, and part of the explanation for them can be found in Kant's perspective on the illegitimate use of methods, particularly the use of the synthetic method in metaphysics on matters that are not permitted. The other part of the explanation concerns the place and role of *definitions* in mathematics and philosophy. These two aspects are connected: the correct use of methods depends on how *definitions* are properly formulated and applied in each of the two domains, mathematics and philosophy. The "middle way", primarily for the benefit of transcendental philosophy, is to bring the two disciplines together indirectly, by investigating the *possibility* of the outcomes of the pure sciences within the framework of later transcendental philosophy. (We will see the connection between the themes of the *Prolegomena* and the discussion of the "experiment of pure reason" in part III of this essay).

Explicitly, Kant has long appreciated the *synthetic* character of the mathematical method, but with the reiteration that this method cannot be imitated as such in philosophy, as many philosophers mistakenly attempted. Although the synthetic method fulfills its full potential in mathematics, where it is naturally situated and operates, the idea of borrowing and adapting the synthetic method into his transcendental philosophy was a constant pursuit for Kant. In many works, he emphasizes the priority that the synthetic method used in mathematics holds over other methods, primarily due to its demonstrative and foundational power, which is not the case with the analytic method. This synthetic character, secondly, consists in the absence of appeal to data and sources that might be suspected of being contingent, whether in terms of experience, observation, or pure speculation.

The privileged appreciation of the synthetic method, which he developed throughout both the pre-critical and critical periods, can also be found in his work *The Only Possible Argument*[44]. I will attempt to elaborate on the points above in order to offer a clarification of the problem of the "imitation" of the mathematical method in philosophy/metaphysics within the context of our broader discussion.

In *The Only Possible Argument*, Kant seems to no longer appreciate metaphysics in relation to geometry in the same way as he did in *Physical Monadology.* Here Kant states that "The mania for method and the imitation of the mathematician, who advances with a sure step along a well-surfaced road, have occasioned a large number of such mishaps on the slippery ground of

[44] Imm. Kant, *The only possible argument in support of a demonstration of the existence of God*, in *Theoretical Philosophy 1755–1770*, pp. 107–201.

metaphysics."[45] As I have already mentioned, I believe, unlike Carson, it should be clarified that this does not refer to a principled limitation on the use of the mathematical method in philosophy. This idea was also valid in the 1760s, when Kant seemed to state with conviction that mathematics and philosophy are clearly distinct and cannot borrow each other's methods (synthesis for mathematics, and analysis for philosophy[46]). In this respect, in a statement from the *Prize Essay*[47], Kant's objections to the metaphysicians' erroneous use of the synthetic method align with the same idea, as "the differences which are to be found between cognition in mathematics and cognition in philosophy are substantial and essential"[48]. Kant states in the *Announcements* that he himself began at some point to proceed according to such a scheme (presumably analytic): "For some considerable time now I have worked in accordance with this scheme. Every step which I have taken along this path has revealed to me both the source of the errors which have been committed, and the criterion of judgment by reference to which alone those errors can be avoided, if they can be avoided at all."[49]

Since the *Prize Essay* had just been published a year earlier, it is clear that Kant is also referring here to how he conceived the method of philosophy in the *Prize Essay*. Therefore, these claims should not be taken *tale quale*, because in the *Prize Essay* the passages from the later work referenced in *Announcements* actually support the opposite: either there are no such essential differences, or for a future project, these differences should not exist[50], and the description of the two perspectives – mathematics and metaphysics – are similar in their synthetic character.

Thus, only by considering the entire path of Kant's engagement with these methods, culminating in what he develops in the B *Critique* as the "experiment of

[45] *Ibidem*, p. 117 [2:71].

[46] "I have sought to show in a short and hastily composed work* that this science has, in spite of the great efforts of scholars, remained imperfect and uncertain because the method peculiar to it has been misunderstood. Its method is not *synthetic*, as is that of mathematics, but *analytic*." (Imm. Kant, *M. Immanuel Kant's announcement of the programme of his leaures for the winter semester I765–766* (*Announcement*), in *Theoretical Philosophy 1755–1770*, p. 294 [2:308]. Kant also refers here* to the *Prize Essay* (Reflection I, AK 2:276–83), where he also stated that the two methods are distinct.

[47] Imm. Kant, *Prize Essay*, p. 255 [2:283].

[48] *Ibidem*, p. 256, [2:284].

[49] Imm. Kant, *Announcements*, in *Theoretical Philosophy 1755–1770*, p. 294 [2:308].

[50] Imm. Kant, *Prize Essay*: "Accordingly, metaphysics has no formal or material grounds of certainty which are different in kind from those of geometry. In both metaphysics and geometry, the formal element of the judgements exists in virtue of the laws of agreement and contradiction. In both sciences, indemonstrable propositions constitute the foundation on the basis of which conclusions are drawn. But whereas in mathematics the definitions are the first indemonstrable concepts of the things defined, in metaphysics, the place of these definitions is taken by a number of indemonstrable propositions which provide the primary data. Their certainty may be just as great as that of the definitions of geometry. They are responsible for furnishing either the stuff from which the definitions are formed, or the foundation on the basis of which reliable conclusions are drawn. Metaphysics is as much capable of the certainty which is necessary to produce conviction as mathematics. The only difference is that mathematics is easier and more intuitive in character.", p. 269 [2:296].

pure reason", can we speak both of the various ways he approached this important aspect of his transcendental philosophy over time and of certain constants or latent ideas in his thinking, which are refined and fully presented in the mature work of the B *Critique*. As I have previously stated, I will attempt to show that it cannot be argued that, in his early writings, Kant limits the use of the synthetic method in (transcendental) philosophy as a matter of principle. On the contrary, as seen in the fragment from note 50, the differences between mathematics and philosophy do not preclude following the synthetic pattern of the mathematical method. What truly matters is this pattern, not the *mere* imitation of the method, which is, in any case, not effectively possible (mathematics operates in a different language, and its concepts have a different referential and probative context, as Kant himself elaborates in the same work). This includes the issue of the place and role of definitions in mathematics and philosophy, which we will discuss further below. From this perspective, I diverge from Emily Carson's perspective.

In contrast to the nuance I intend to make and which I consider important, E. Carson proceeds differently in her research, making use of a certain "mix" of Kant's statements from two different works regarding the "imitation" of the mathematical method to emphasize the dichotomy between mathematics and philosophy regarding the first's method being imitated by the latter[51]. Presented this way, although they seem somewhat coherent, they distort what Kant actually said, and implicitly the meaning of these fragments. Therefore, especially considering the many nuances sometimes confusingly expressed by Kant, I think it is useful to look at these fragments in their entirety, rather than paraphrasing from other works so that they seem to refer to the same thing.

In Carson's text, referring to what Kant might have said, we find the following (with my additions in parentheses): "nothing, [...], has been more damaging to philosophy than its attempt to imitate the method of mathematics (1764 – *Prize Essay*, 2:283). The application of the mathematical method in philosophy, however, has allowed the latter to attain heights to which it otherwise could not have aspired (1763 – "Preface" to *Attempt to Introduce…*, 2:167). But instead of thus turning the insights of mathematics to its own advantage, metaphysics has 'armed itself against them': where it might, perhaps, have been able to gain secure foundations on which to base its reflections, [...]."[52]

In fact, as I pointed out above, in Carson's paraphrase we are dealing with two fragments from two different works, where each of the sentences was completed with something different from what appears in the compiled fragment by Carson. The first sentence is completed with what I have noted in italics below: "nothing has been more damaging to philosophy than mathematics, and in particular the imitation of its method *in contexts where it cannot possibly be employed*" – as can be seen, the last clarification by Kant here is important; it refers

[51] See E. Carson, "Kant on the Method of Mathematics", p. 631.
[52] *Ibidem*.

to contexts in which it cannot be employed, not in general. The second sentence talks about the benefits of applying mathematics to philosophy, but in Kant's text, it is not about metaphysics, rather it concerns the application of the method in certain "parts" of philosophy: "But the parts of philosophy to which I am referring are only the insights of physics." The third sentence in Carson's paraphrase introduces the word "but" to force the equivalence between what was said earlier about philosophy and what is now being said about metaphysics, as though they were the same thing. "But instead of thus turning the insights of mathematics to its own advantage, metaphysics has 'armed itself against them'." But Kant refers here strictly to the "science of metaphysics" distinct from philosophy, in a sense I will clarify below. Now I will present the two passages in full from Kant:

> nothing has been more damaging to philosophy than mathematics, and in particular the *imitation* of its method in contexts where it cannot possibly be employed. The *application* of the mathematical method in those parts of philosophy involving cognition of magnitudes *is something quite different*, and its utility is immeasurable. *Prize Essay* 2:283 [my emphsys on *is*..]

The second fragment:

> As for metaphysics, this science, instead of turning certain of the concepts or doctrines of mathematics to its own advantage, has, on the contrary, frequently armed itself against them. And where it might, perhaps, have been able to gain secure foundations on which to base its reflections, it is to be seen trying to turn mathematical concepts into subtle fictions, which have little truth to them outside the field of mathematics. It is not difficult to guess which side will have the advantage if two sciences enter into a dispute with each other, where the one excels all others in certainty and distinctness, while the other has only just started out on the path to these objectives.[53]

Resuming the initial question: what is this method? Could it be the analytic method, as Kant briefly states in the *Announcement* that he would have tried in the *Prize Essay?* Could it even be the synthetic method? To clarify these aspects, I will now refer to some of his statements in which he distinguishes several interpretations of philosophy and metaphysics (from the *Prize Essay*), to understand exactly what the he means when he says that philosophy cannot imitate the method of mathematics, but has benefited from its *application* to philosophy.

Kant states that:

> Metaphysics is nothing other than the philosophy of the fundamental principles of our cognition. Accordingly, what was established in the preceding reflection about mathematical cognition in comparison with philosophy will also apply

[53] Imm. Kant, *Attempt to introduce the concept of negative magnitudes into philosophy*, in *Theoretical Philosophy 1755–1770*, p. 207 [2:168].

> to metaphysics. We have seen that the differences which are to be found between cognition in mathematics and cognition in philosophy are substantial and essential.[54]

The differences between philosophy and mathematics discussed at length by Kant in the first reflection also apply to metaphysics. However, if we think systematically, we can see that philosophy is more comprehensive than metaphysics: we have philosophy in general, which would include metaphysics, but also the "parts" of philosophy, where mathematics can be applied to the former, that is, to the parts that share a common object with physics, such as "intuitions of physics"; for example, those related to concepts of magnitude, or those concerning data that can be provided by mathematics. As for metaphysics, here Kant defines it in the sense of the critical project, as "the philosophy of the fundamental principles of our cognition", that is, the science of the principles of cognition, not in the sense of "the science of the first principles". This distinction allows us to understand that there is compatibility between mathematics and metaphysics/philosophy in terms of methods, especially since Kant states that "It is not difficult to guess which side will have the advantage if two sciences enter into a dispute with each other, where the one excels all others in certainty and distinctness, *while the other has only just started out on the path to these objectives*."[55] [my emphasis].

Based on this last clarification, we can argue that there are two approaches to metaphysics: "traditional" metaphysics that already had a tumultuous history due, among other reasons, to an inappropriate use of the synthetic method of mathematics; and a different kind of metaphysics/philosophy still in formation that Kant was concerned with, and is of interest here. According to this second approach, there is no principled limit to combining the methods of mathematics and metaphysics (analysis and synthesis). In some places Kant argues that the use of the analytic method should clarify some aspects before the synthetic method is applied in metaphysics, as in other places the synthetic character is clearly present from the outset, as applied to some of the clear and distinct elements that are taken synthetically even in metaphysics – elements that he describes as equivalents of those in mathematics.

Kant will shed more light on the issue we are dealing with now through analyses where he develops the discussion about *definitions* and their significance in mathematics and philosophy, and some of the important elements from here will be found much later in the two editions of the *Critique* (for example, at A 727/B 755). In the *Prize Essay*, one of the works more dedicated to the discussion of methods in mathematics and philosophy, we find that the possibility and manner of using the two methods in mathematics and philosophy are related to the place, role, and

[54] Imm. Kant, *Prize Essay*, 2:283.

[55] Imm. Kant, *Attempt to introduce the concept of negative magnitudes into philosophy*, in *Theoretical Philosophy 1755–1770*, p. 207 [2:168].

status of *definitions*. I think that the meanings in which Kant understands, in his 1760s's works, the relationship between the method of mathematics and philosophy and metaphysics (in both senses), as well as the distinction between *imitation* and *application* should be taken into consideration in their *interrelationships*, for a clearer understanding of the problem of relating methods in and between the two disciplines.

I will now focus on the problem of definitions and the *sense* in which I believe the synthetic method should and even is "imitated" in Kant's philosophy, initially timidly, in his early works such as the *Prize Essay*, as well as the way in which the mathematical method was incorporated into the critical program. This will be the focus of my analysis, after which I will return, in part III, to how Kant incorporates the two methods in the B *Critique*.

The reason why Kant denounces the imitation of the method of geometry in philosophy and metaphysics is not a principled incompatibility, as I have shown earlier, and as might be easily interpreted from some of Kant's expressions, but the difference in *the clarity and distinctness of the concept or primary elements from which one begins*. Although he appreciates the method of geometry, Kant does not recommend imitating the mathematician's method *when starting with definitions in philosophy*. The clarity and distinctness of definitions in philosophy are inferior to those of geometric definitions. Furthermore, analysis, which is more suitable for clarifying concepts in philosophy, does not provide *any* "foundational" character, being a mere clarification of more obscure concepts. This is how apparently contradictory Kantian fragments – such as when he states that much time must pass before the synthetic method can be applied in philosophy[56], and when he argues that the use of the analytic method is necessary first to ensure that we do not have anything unclear before proceeding synthetically – should be understood.

In mathematics, Kant says, I start with the definition of my object, for example, of a triangle or a circle, whereas in metaphysics, I can never start with a definition[57]. In metaphysics, the definition is almost always the last thing I come to know; on the contrary, in mathematics, we first have the definitions and then the concepts. In metaphysics, concepts are given from the outset, but in a confused manner. What the philosopher must do is seek a *distinct*, *complete*, and *determined* concept[58]. Kant explains these differences by saying that the two sciences arrive at their definitions through different methods. The synthetic in the mathematical method must be understood in the sense that definitions are obtained through the "arbitrary combination of concepts". The concept thus defined is not given before the definition, but rather emerges with the definition. In philosophy, concepts are

[56] „Metaphysics has a long way to go yet before it can proceed synthetically. It will only be when analysis has helped us towards concepts which are understood distinctly and in detail that it will be possible for synthesis to subsume compound cognitions under the simplest cognition, as happens in mathematics." (*Prize Essay*, 2:291).

[57] *Prize Essay*, [2:283].

[58] *Ibidem*.

always given in some way, but in a confused or insufficiently determined manner. What the philosopher does is to discover, through *analysis*, the characteristic features of the confused concept in order to arrive at a complete and determined concept, that is, a definition. Although Kant attributes many errors in philosophy to the failure to recognize this difference between philosophy and mathematics, he will constantly refer to the synthetic *character* of the mathematical method. The synthetic method in mathematics should not be imitated in philosophy – I would clarify – *at least not with respect to definitions* ***as such***.

I will address now some fragments from the *Lectures on Logic* where Kant refers to the conditions of a definition in mathematics, a necessary step to clarify the meaning and significance in which Kant, through these conditions, establishes not only a theoretical status of the definition, but also characterizes the synthetic (a priori) nature of the mathematical method and what distinguishes the status of exemplary certainty that this discipline has inspired, not only in general but even for philosophy.

A definition in mathematics presupposes that a concept be *distinct*, *complete*, and *precise*. A concept of a thing is 1) *distinct* insofar as a) there is clarity of signs in this concept[59], that is, to the extent that one is aware of the signs contained in the concept; furthermore, b) those signs must be *clear foundations for the knowledge of the thing*, meaning that the signs of the *definition* are distinct from those of the *definitum*[60]: a tautology, for example, is not a definition because the signs in the supposed definition are not distinct from those of the definitum, since "for what merely says the same thing names no ground for me"[61]. A concept is 2) *complete* (or completely distinct) when a) the signs are sufficient to know, first, a) the difference of the definitum from all other things, and secondly, b) its identity with other things. Finally, a definition is 3) *precise* when none of the signs in the definition is already contained in another: for example, "a body is matter that is extensible and divisible" is not precise, because the sign of divisibility is found in the sign of matter, thus it is redundant[62].

Therefore, these conditions are for definitions in mathematics, the only domain in which they are accepted because they are definitions of "arbitrary concepts": one is aware of each of the signs included in the concept because one has placed them there in defining the concept, and one can most easily be conscious of that which one has oneself invented[63]. The definition is complete because the mathematician considers everything that is sufficient to distinguish the thing from all others, because [he] is not a thing found outside himself, which he

[59] Imm. Kant, *The Bloomberg Logic*, in *Lectures on Logic*, p. 92 [120].

[60] *Ibidem*, pp. 212–213 [265].

[61] *Ibidem*, p. 213 [265].

[62] For a more comprehensive synthesis, see E. Carson ("Kant on the Method of Mathematics", pp. 634–636).

[63] Imm. Kant, *The Bloomberg Logic*, in *Lectures on Logic*, p. 121 [153].

has partially known according to certain determinations, but rather a thing in his pure reason, with which he arbitrarily thinks and to which he attaches certain determinations, by means of which he intends the thing to be distinguishable from all other things[64].

Indeed, if the thing defined is first given through the definition, then the definition is complete. Whatever satisfies the definition of a triangle is a triangle, says Kant. Precision is the only aspect in which the mathematician can err, "but this is not an error, but merely a mistake, and from this we see again what a great advantage a mathematician has"[65], according to Kant.

Kant shows that both empirical and philosophical concepts share difficulties that are not encountered in definitions in mathematics. In the case of empirical concepts, even though they are given, in order to make them distinct, it is necessary to enumerate all the signs associated with the defined term. However, it cannot be known for certain that these signs are sufficient to definitively distinguish the concept itself from others. At best, a comparative completeness can be achieved, when the signs are sufficient to distinguish the concept from everything that has been known up to that point through experience. This completeness is specific to simple description. Similarly, philosophical concepts are given to some degree as confused, and the philosopher cannot be certain that all the necessary signs for complete understanding[66] have been identified, leaving open the possibility that unknown signs may be relevant. Thus, philosophical concepts, like empirical ones, cannot be completely defined, and any definition remains uncertain. This is why Kant will later say in the *Critique of Pure Reason* (B edition) that, in the case of the pure concepts of the intellect, a deduction of them is needed – this deduction is nothing other than a specific combination of methods from mathematics, adding the idea of the synthetic a priori and the conditions of possibility (the transcendental). As for metaphysics, unlike mathematics, where all proofs refer to the definition and are fully accessible to our view in terms of evidence, Kant says that it "still has a long way to go before it can proceed synthetically".

Kant distinguishes between objective certainty and subjective certainty. In both types of certainty, mathematics is superior to metaphysics: objective certainty consists in the fact that in mathematics one proceeds synthetically, meaning that it can be stated with certainty that what was not intended to be represented in the object through the definition is not contained in that object (since the concept "comes into existence" through the definition – it has "no other meaning"); while subjective certainty consists in considering mathematical knowledge as universal under concrete signs. Thus, the sufficiency and necessity of the characteristics are guaranteed.

[64] *Ibidem*, p. 97 [125].
[65] *Ibidem*, p. 216 [269].
[66] *Ibidem*, p. 96 [124].

In philosophy, on the contrary, it can be argued that a characteristic does not belong to a concept simply because that characteristic is not observed, as philosophy always considers its knowledge as universal *in the abstract*, existing *alongside* signs. Therefore, philosophy and metaphysics are more uncertain "in their definitions"[67].

Indeed, as Carson points out, because the signs in mathematics are "sensible means of knowledge", we can know that no concept has been omitted and that the rules have been followed with the degree of certainty as when "the thing is placed before the eyes"[68]. I would add that the signs in mathematics have a different epistemological status: they are "proofs", unlike the "words" in philosophy. The signs in philosophy, the words, only point to the universal concepts they signify in the abstract. Constant attention is required to be aware that the rules have been followed correctly[69].

Let us summarize E. Carson's conclusion[70] regarding the mathematical method described in the *Prize Essay*. It begins with a few fundamental concepts that mathematicians neither can nor should define, such as magnitude in general, unity, plurality, and space, as well as a small number of indemonstrable propositions deemed "immediately certain" or "presumed to be true". These include principles such as the whole being equal to the sum of its parts and the uniqueness of a straight line between any two points. Synthesis is the process through which additional concepts are constructed from these foundational elements, primarily through arbitrary combination. The mathematician then derives further propositions from these complex concepts, along with the foundational propositions. However, in proofs and inferences, the mathematician does not directly engage with the objects themselves or their universal concepts but rather works with their symbolic representations. The greater objective certainty in mathematics stems from the synthetic method, which ensures that definitions exclude unintended representations – allowing one to affirm with confidence that what is defined is precisely what is present in the object, leaving no room for misinterpretation. Meanwhile, the heightened subjective certainty derives from the use of sensible signs.

If above we had an overview of the mathematical method from the *Prize Essay*, it is now time to consider the two rules that Kant formulated here – rules that would govern the method by which the highest possible degree of metaphysical certainty can be achieved. The discussion will continue by following Kant's own indications regarding this method, which refers to its similarity with Newton's

[67] In this respect, see E. Carson's argumentation ("Kant on the Method of Mathematics", pp. 640–641).

[68] I will return below to this Kantian phrase, which I consider important in the context of the present discussion.

[69] E. Carson, "Kant on the Method of Mathematics".

[70] *Ibidem*, p. 641.

method (in intimate correlation with the method of geometry[71]). We know that, even in *The Only Possible Argument...*, Kant expressed his admiration for Newton's method, but did so at the expense of the method of metaphysicians[72].

The first and most important rule is that, in philosophy: I) *one should not begin with definitions*, but rather begin by *carefully examining what is immediately certain in the object of study, even before having a definition of it*. Once what is immediately certain in the object of research has been established, *conclusions can be drawn from it*; one will reach judgments about objects that are true and completely certain; not only is it not necessary to arrive at a definition, but it is not even expected until, eventually, it will naturally present itself based on the most certain judgments. The second rule is that II) *one should distinguish between judgments that have been immediately formulated about the object* and that refer to what was initially encountered in that object with certainty. Once it has been firmly established that none of these judgments is contained within another, *these judgments should be placed at the beginning of the research as the foundation of all deductions, just as axioms are in geometry*, Kant adds[73].

A brief comment on this: with the exception of the necessity to begin with definitions, in both rules we actually have a "synthetic beginning" of the philosophical method, similar to what the definition in mathematics entails; the step of clarification specific to analysis is almost nonexistent here (it could be attributed to the distinction between judgments through comparison, etc., in the second rule). Furthermore, Kant explicitly states that these certain judgments must be placed at the beginning of the research "just like axioms in geometry" – exactly as in the synthetic method. Such an arrangement leads us to consider the evaluation of the synthetic method in A *Critique of Pure Reason*, an evaluation also present in *Prolegomena*. (I will return to these aspects later.)

It worth noticing that, after listing the above, for exemplifying the "true method of metaphysics", Kant immediately refers to Newton's method, describing a passage from *Opticks* almost in the terms of his later transcendental philosophy. Here is the excerpt in which Newton's method resonates up to identification with what the "true method of metaphysics" should be:

> *The true method of metaphysics is basically the same as that introduced by Newton into natural science* and which has been of such benefit to it. Newton's method maintains that one ought, on the basis of certain experience

[71] Newton's method is appreciated in contrast to that of the metaphysicians with respect to the problem of action at a distance. E. Carson argues in her text that here Kant positively appreciates Newton's method as proceeding on the basis of empirical observation and mathematical inference to "indubitably correct" and "self-evident" conclusions; while the metaphysicians begin with surreptitious definitions, and therefore "have no sound reason to object to the idea of immediate attraction at a distance" (see E. Carson, "Kant on the Method of Mathematics", p. 642).

[72] See Imm. Kant, *The Only Possible Argument*, p. 180 [2:139].

[73] See Imm. Kant, *Prize Essay*, p. 258 [2:286].

> and, if need be, with the help of geometry, to seek out the rules in accordance with which certain phenomena of nature occur. Even if one does not discover the fundamental principle of these occurrences in the bodies themselves, it is nonetheless certain that they operate in accordance with this law. Complex natural events are explained once it has been clearly shown how they are governed by these well-established rules. Likewise in metaphysics: by means of certain inner experience, that is to say, by means of an immediate and self-evident inner consciousness, seek out those characteristic marks which are certainly to be found in the concept of any general property. And even if you are not acquainted with the complete essence of the thing, you can still safely employ those characteristic marks to infer a great deal from them about the thing in question.[74] [my emphsys]

Even more interesting is the fact that, in an earlier work (*The Only Possible Argument...*), Kant's admiration arises *solely* in connection with Newton's method (regarding attraction at a distance) as proceeding on the basis of "empirical observation and mathematical inference"[75], but in contrast to that of metaphysics, where a definition is eventually formulated according to some taste.

The nuances I aim to highlight reveal two distinct approaches to metaphysics: one rooted in traditional metaphysics, separate from Kant's later critical project, and another whose foundations he was already laying as early as the 1760s, if not earlier. Furthermore, we observe certain shifts and emphases in Kant's position throughout his pre-critical period. What remains consistent, however, is his distinctive appreciation for the mathematical (synthetic) method and Newton's approach. In certain pre-critical writings, Kant does not outright reject the application of the mathematical method to philosophy but instead confines it to specific contexts or leaves it open to debate – contrasting with other instances where his tone regarding this dichotomy is more definitive. In light of this, and for the subsequent analysis, I found it necessary to introduce greater nuance.

III. METHOD IN THE CRITICAL PERIOD: A, *PROLEGOMENA*, B

To outline how Kant incorporated the two mathematical methods into his critical project in the B edition, I will first establish several key points that will guide this section of my essay.

The prohibition against imitating the mathematical (synthetic) method during the pre-critical period primarily targets traditional metaphysics, not the "true metaphysics", as I have argued above and will revisit below. Kant's attempt to "imitate" Newton's mathematical method in the *MFNS,* as he himself acknowledged

[74] *Ibidem*, p. 259 [2:288].
[75] Imm. Kant, *The Only Possible Argument*…, p. 181 [2:139].

in the "Preface" to this work[76], substantiates this claim. However, as we shall see, Kant's adoption of these mathematical methods, as well as Newton's, in his critical project was dynamic and adapted, rather than direct or unmodified.

A clarification, somewhat tangential to this context but pertinent to the fourth section of my text, is that Kant did not explicitly or systematically emphasize the aspects of his critical project concerning the adoption of these *mathematical* methods – at least not in the same manner as he did during the pre-critical period. This indicates the *heuristic* character of their lineage within his mature transcendental philosophy (as articulated in the B edition of the *Critique*). This heuristic character becomes evident under a particular interpretation, which I have advanced in some previously published works, where these subtler aspects gain coherence and validity within the critical program – a point I will return to in this essay's conclusion.

What we take from the first part's final analysis is that Immanuel Kant never give up the idea of introducing and correctly/adaptively employing the synthetic (and analytic) method(s) from mathematics in philosophy. Kant frequently criticized the inappropriate use of the mathematical synthetic method in metaphysics, as practiced by philosophers influenced by the intellectual trends of his time. As discussed earlier, the coordinates of *definition* in mathematics and philosophy, along with the "two rules" concerning the method of "true metaphysics", address precisely this issue: given the distinct nature of philosophical inquiry, it requires the condition of *not starting with definitions*, unlike in mathematics. However, this does not imply a rejection of the *synthetic* (or, as Kant would later call it, "general") nature of the method. That is, philosophy must also begin with something of high evidential value, regarded as certain, and analyze it into its elementary consequences. Crucially, these "elementary consequences" must themselves serve as conditions for *other* elements. As noted earlier, it is essential that the outcome of synthesis – the *a priori* combination of initial primary elements assumed as principles – is not merely tautological. Instead, this result must function as *a condition of possibility for other phenomena*, embodying a foundational and generative character.

The "analytic" aspect of metaphysics during the pre-critical period served primarily to ensure that nothing was assumed to be certain or foundational – principles of the first order – without sufficient justification. From this perspective, Kant discusses in the pre-critical period the method of analysis, which should precede synthesis. As we have seen, Kant remarked that it would take a long time before philosophy could begin synthetically (I would add here: until his mature critical project in B *Critique*). However, the demonstrative force lies with the synthetic method, not the analytic one. Thus, in the order of demonstration, the

[76] See Imm. Kant, *Metaphysical Foundation of Natural Science*, translated and edited by Michael Friedman, Cambridge, Cambridge University Press, 2004. The well known reference is from the "Preface" (478), pp. 13–14.

only method that truly matters and "must take precedence" is the synthetic one. Here, the starting point must introduce purely a priori and "simple" objects, "like a principle" – whether this be the *intellect* and *sensibility* – subsequently decomposed into their fundamental elements (the forms of space and time, the categories), which then serve as foundation for the possibility of *other* a priori knowledge.

Below, I will summarize how Kant incorporated these two mathematical methods into his critical program, distinct from their logical sense and adapted to transcendental philosophy.

In the "Preface" of A *Critique* Kant argues that by critique he did not understanded "....a critique of books and systems, but a critique of the faculty of reason in general, in respect of all the cognitions after which reason might strive independently of all experience, and hence the decision about the possibility or impossibility of a metaphysics in general, and the determination of its sources, as well as its extent and boundaries, all, however, *from principles*" [my emphasis][77]. By analytic, Kant understands the analysis or descomposition of the entirety of our a priori cognition into the elements of the pure cognition of the understanding. It is concerned with the following: 1. That the concepts be pure and not empirical concepts. 2. That they belong not to intuition and to sensibility, but rather to thinking and understanding. 3. That they be elementary concepts, and clearly distinguished from those which are derived or composed from them. 4. That the table of them be complete, and that they entirely exhaust the entire field of pure understanding.[78]

It is quite clear that here Kant distinguishes the analysis of understanding from the analytic method in mathematics (see the first section), and subsequently explicitly distinguishes it from the logical treatment, as follows: "an analytic of concepts not their analysis [*zergliedern*], or the usual procedure of philosophical investigations, that of analyzing the content of concepts that present themselves and bringing them to distinctness, but rather the much less frequently attempted analysis [*Zergliederung*] of the faculty of understanding itself, in order to research the possibility of a priori concepts by seeking them only in the understanding as their birthplace and analyzing its pure use in general; for this is the proper business of a transcendental philosophy; the rest is the logical treatment of concepts in philosophy in general. We will therefore pursue the pure concepts into their first seeds and predispositions in the human understanding, where they lie ready, until with the opportunity of experience they are finally developed and exhibited in their clarity by the very same understanding, liberated from the empirical conditions attaching to them"[79].

Given the discussions in the first part, it becomes obvious that in the A *Critique*, Kant primarily employs the synthetic method. The synthetic method in

[77] Imm. Kant, *Critique...* (A XII).
[78] *Ibidem*, B 91.
[79] *Ibidem*, A 66/B 91.

mathematics assumes that the starting point must stem from something akin to first principles (Pappus). Here, it is clear that beginning *from principles* "[*Principien*]" signifies the synthetic or progressive method, as the foundational character of the categories also derives from this same method. In addition to the well-known reference in the *Prolegomena*, where Kant explicitly states that, unlike the *Prolegomena*, where he employed the analytic method, he used the synthetic method[80] in the A *Critique*, I would introduce another reference here. This reference has been less frequently addressed in exegesis but is significant within the context of our essay. I will provide the original German text in a footnote to preserve the precise formulation, which has often been overlooked by English translators. This omission is fundamental to understanding the origin – dating back to the pre-critical period – of one of the meanings of the synthetic method within the critical period, beginning with the first edition of the *Critique*.

> This sort of investigation will always remain difficult, for it includes the metaphysics of metaphysics. Yet I have a plan in mind according to which even popularity might be gained for this study. However, this plan could not be carried out initially, for the foundations needed cleaning up, particularly because the whole system of this sort of knowledge had to be exhibited[81] in all its articulation. [AA 10:269].

It is obvious that in this letter (to Marcus Herz after May 11, 1781), Kant uses the same expression *Augen gestellt werden musste*, (which should have appeared in this equivalent expression in English translation: "had to be brought before one's eyes"), as in the pre-critical period. Here, as in many other instances, Kant used this expression to describe a specific feature of the synthetic method, stating,

[80] "In the *Critique of Pure Reason* I worked on this question [*Is metaphysics possible at all?*] *synthetically*, namely by inquiring within pure reason itself, and seeking to determine within this source both the elements and the laws of its pure use, according to principles. This work is difficult and requires a resolute reader to think himself little by little into a system that takes no foundation as give except reason itself, and that therefore tries to develop cognition out of its original seeds without relying on any fact whatever. *Prolegomena* should by contrast be preparatory exercises; they ought more to indicate what needs to be done in order to bring a science into existence if possible, than to present the science itself. They must therefore rely on something already known to be dependable, from which we can go forward with confidence and ascend to the sources, which are not yet known, and whose discovery not only will explain what is known already, but will also exhibit an area with many cognitions that all arise from these same sources. The methodological procedure of *Prolegomena*, and especially of those that are to prepare for a future meta-physics, will therefore be *analytic*." (*Prolegomena*, [4:275]).

[81] Imm. Kant, Letter to Marcus Herz, After May 11, 1781. This is the original text from *Kants Briefe*, Ausgewählt und herausgegeben von F. Ohmann, I Band, Leipzig, Erschienen im Insel-Verlag, Leipzig, pp. 91–95: "Schweer wird diese Art Nachforschung immer bleiben, denn sie enthält die Metaphysik von der Metaphysik, und gleichwohl habe ich einen Plan im Gedanken, nach welchem sie auch Popularität bekommen kann, die aber im Anfang, da der Grund aufzuräumen war, übel angebracht gewesen wäre, zumal das Ganze dieser Art der Erkenntnis nach aller seiner Artikulation vor *Augen gestellt werden musste* […]." [my emphasis] (AA X 269).

for example, that "only in which the coherence and clarity of the reasoning are presented with complete precision before the eyes". Kant frequently employed this phrase during the pre-critical period, particularly to illustrate the evidence-based nature of the synthetic method: the clarity of geometric demonstrations, the rigor of mathematical reasoning, and *the means* of verifying the precision of methodological sequences in algebra. In the absence of construction in pure intuition – a pivotal element of transcendental philosophy that became possible only after the development of the theory of pure intuition in the *Dissertation* – this expression generally referred to what ensured evidence, serving *as its accompaniment*. *Augen gestellt werden musste*, therefore, was used to emphasize the evidential character inherent in both disciplines (metaphysics and mathematics), later adapted in the critical period, at times in association with the description of the method of construction in pure intuition (as Kant discusses the distinctiveness of mathematics in A 735/B 763), or when Kant explains the mechanism of a priori synthesis underlying arithmetic enumeration. More broadly, however, it indicated what accompanies evidence (e.g., A 103, A 108, B 20, among others).

Returning to the quoted passage, we can conclude that the *Critique* operates as a "metaphysics of metaphysics", establishing its foundations, where the *synthetic* method is deemed essential. The use of the expression (*Augen gestellt werden musste*) reveals not only the precedence of the synthetic method over the analytic but also the meta-theoretical and transcendental *a priori* foundational level at which Kant's theory operates within the *Critique*.

However, the major transformation of the mathematical method in the context of the critical program is found in the B Preface, in the formulation of the "experiment of pure reason". This formulation embodies both an "adapted variant" of Newton's method, which Kant had praised in the *Prize Essay* and many other pre-critical works, as well as the two mathematical methods, reconfigured in a transcendental framework. The "adapted variant" in the A edition was achieved through the combination of synthetic and analytic methods, shaped by the model of Newton's constructive mathematical method – yet as Kant interpreted this model, closely aligned with his view of Newton's method from the *Prize Essay*. While Newton acknowledges the mathematical origins of these two methods (composition and analysis), Kant appropriates and subsumes them under a general method that resonates with what he terms "analytic", i.e., the analysis of understanding (as presented in the A edition and discussed earlier – distinct from the logicians' analysis or the mathematicians' analytic method as found in Pappus or Polya).

> This experiment of pure reason has much in common with what the **chemists** sometimes call the experiment of **reduction**, or more generally the **synthetic procedure**. The **analysis of the metaphysician** separated pure a priori knowledge into two very heterogeneous elements, namely those of the things as appearances and the things in themselves. The **dialectic** once again

> combines them, in unison with the necessary rational idea of the unconditioned, and finds that the unison will never come about except through that distinction, which is therefore the true one.

Up to this point, we have outlined the hypothesis of this "experiment", which can be summarized as follows: how can we coherently and foundationally explain the possibility of synthetic *a priori* knowledge, assuming two variants – either it is oriented toward objects, or alternatively, they are determined by our cognitive faculties? Kant argues (B XXII n.) that he has demonstrated apodictically (I would also add *synthetically* in the A *Critique*) that the first variant is inconsistent, while the second has been validated, thus establishing it as the hypothesis for this experiment. In other words, this hypothesis can be rephrased using another Kantian phrase: "that we only know phenomena as objects and not things-in-themselves".

It is now interesting to see that Kant's understanding and explanation of the similarity, to the point of identity, between Newton's mathematical method and that of "true metaphysics" in the *Prize Essay* actually involves a transcendental understanding of this method, which will align with the method of the later critical project in the B edition of the *Critique*. In other words, I will show how the goal set out in the *Prize Essay* (the transcendental interpretation of Newton's method, as Kant describes it here) for "true metaphysics" was embodied in the project of B *Critique* as the "experiment of pure reason". The method of this experiment will unify this transcendental understanding of Newton's method (the goal) with what Kant states about the method of the pure reason experiment in the B Preface (method borrowed from physicists and chemists).

In this sense, I will present the original text from Newton's *Opticks*, so that the comparison between the meaning here and how Kant intends to project this method through its similarity to the "true metaphysics" will highlight what truly interested Kant and what he retained from Newton's method for his later critical project. In the next step, I will show that the meaning of the synthetic that Kant retained in his interpretation of Newton's method as it appears in the *Prize Essay* (as similar to that of the "true metaphysics") is precisely that of the method of the experiment of pure reason in the B Preface (which retains some of the characteristics from mathematics[82] at the level of its facets – synthetic and analytic). More specifically, I attempt to show in the following that in the B *Critique* Kant understood Newton's method as "generally synthetic", and that he related it to the method of physicists and chemists (in the B Preface) within the framework of the "experiment of pure reason". I also try to show how Kant uses both the synthetic and analytic mathematical methods in combination and to

[82] What is characteristic of the synthetic is the fact of starting "from principles" and placing this mode of investigation at the outset of research, along with the *a priori* and foundational status of its results – so that what is thus investigated and obtained serves as ground.

emphasize their participation within the framework of the "experiment of pure reason".

With the critical project, in which Kant clarified the fundamental role of mathematics in sciences[83], the experiment in sciences is transcendentally projected through both methods (synthetic and analytic) within this experiment of pure reason (developed according to a generally "synthetic" procedure), with meanings closer to those found in mathematics[84].

Here is the fragment from *Opticks*, which Kant most likely had in mind in the *Prize Essay* when he compared the method here with that of the "true metaphysics", stating that they are the same:

> As in Mathematicks, so in Natural Philosophy, the Investigation of difficult Things by the Method of Analysis, ought ever to precede the Method of Composition. This Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction, and admitting of no Objections against the Conclusions, but such as are taken from Experiments, or other certain Truths… By this way of Analysis we may proceed from Compounds to Ingredients, and from Motions to the Forces producing them; and in general, from Effects to their Causes, and from particular Causes to more general ones, till the Argument end in the most general. This is the Method of Analysis: And the Synthesis consists in assuming the Causes discover'd, and establish'd as Principles, and by them explaining the Phenomena proceeding from them, and proving the Explanations.[85]

This is the clearest passage where Newton links the method of physics to the methods of mathematics. As we saw earlier, in his interpretation of this method in the *Prize Essay*, when Kant compares it to the method of "true metaphysics", he ignores "induction", and the experiments become "certain inner experience", "by means of an immediate and self-evident inner consciousness". The method then

[83] "I assert, however, that in any special doctrine of nature there can be only as much proper science as there is mathematics therein." (Imm. Kant, *Metaphysical Foundation of Natural Science*, "Preface", p. 6.).

[84] I am referring here to the "definition" of the transcendental in the B Introduction, which includes the phrase "I call all cognition transcendental that is occupied not so much with objects but rather with our mode of cognition of objects insofar as this is to be possible a priori" (B Introduction, B 25/B 26); to the synthetic character of the deduction in the B edition, which requires, in the deduction of the pure forms of space and time and the categories, starting from simple principles – for example, considering space as "an infinite given magnitude", from which the other characteristics are deduced, leading to its determination as pure intuition and not a concept. This ultimately provides a foundational character not only for the construction of geometry in pure intuition but also for any theoretical (scientific) construction that aspires to relate to an object; or to the "principle of the synthetic unity a priori", from which the categories will be deduced – through metaphysical and transcendental deductions – that will serve, as *principles*, as the foundation for any relation, in general, to objects and to phenomena as objects of possible experience.

[85] Issac Newton, *Opticks*, New York, Dover Publication, Inc., 1952, Book Three, Part I, pp. 404–405.

proceeds to "seek out those characteristic marks which are *certainly to be found in the concept of any general property*" [my emphasis]. And, "even if you are not acquainted with the complete essence of the thing, you can still safely employ those characteristic marks to infer a great deal from them about the thing in question". It is evident and understandable why Kant did not use "induction" here, nor terms referring to scientific experiments, to which Newton explicitly refers. In the domain of metaphysics (the later pure reason), Kant could not include empirical elements, nor could he rely on the experiment of natural science (Newton's physics). However, the use of the latter became possible in a special way only after Kant developed the doctrine of pure a priori intuition and, thus, the theory of construction in pure intuition, specific to mathematics. For, once this doctrine was developed (already in the *Dissertation,* 1770), along with the clarification of the role of mathematics in natural philosophy[86] within the critical project, despite the presence of the empirical in any natural philosophy, thanks to the "pure part" of any such physics, it became much easier for Kant to incorporate the model of scientific experiment into his critical project.

The analogy of the "experiment of pure reason" with the "experiment of physicists" thus becomes possible, finally aligning with the interpretation that I would call "transcendental" of Newton's method from the *Prize Essay*: for, as Kant states in B Preface, because „the propositions of pure reason, [...] admit of no test by experiment with their objects [*Objecte*] (as in natural science): thus to experiment will be feasible only with **concepts** and **principles** that we assume *a priori*" [B XVIII]. The synthetic nature of the method (beginning the experiment a priori and working with concepts and principles) also emerges from these words, but it becomes even clearer in the fragment from the second note regarding the experiment, where Kant explicitly mentions the synthetic nature of the experiment in comparison to the "chemists' experiment", as I have previously indicated: „This experiment of pure reason has much in common with what the chemists sometimes call the experiment of **reduction**, or more generally the **synthetic procedure**."[87]

A historical note here is valuable, in the sense that the analysis conducted by the "chemist" during Kant's time was a "rational" one. The aim was for the analysis to reveal the composition of composite bodies and what made them composite, rather than merely isolating the elements. What was sought were, in fact, the "invisible mechanisms that would generate the order of phenomena", mechanisms formulable with the help of "elements" understood in this sense as "principles of the internal composition of composite bodies"[88]. It is now even easier to observe the ("general") *synthetic* character of the method of the experiment of pure reason from the B edition of the *Critique*. I use the term "general" by paraphrasing Kant,

[86] See note 83.

[87] Imm. Kant, B *Critique*, Preface, XXI.

[88] Richard Westfall, *The Construction of Modem Science*, Cambridge, Cambridge University Press, 1977, Ch. IV, *apud*. I. Pârvu, *Posibilitatea experienţei. O reconstrucţie teoretică a Criticii raţiunii pure*, Bucureşti, Politeia, 2004, pp. 331–332.

because it is only in the B edition that he could fully develop this experiment (which incorporates both methods, analytic and synthetic, but in a sense closer to that found in mathematics).

Thus, we have seen that in the A edition, Kant employed the synthetic method (this is indisputable based on all the elements presented above); similarly, in the *Prolegomena*, it is obvious that Kant employed the analytic method (starting from the sense of regressivity, the sense of the analytic as found in mathematics, but introducing for principles the "conditions of possibility" – principles that underlie the possibility of synthetic a priori knowledge, pure mathematics, and pure physics).

Considering the "order" of these methods – which, as shown in the first section of this essay, can involve the priority of either the synthetic method (followed by the analytic method) or the analytic method (followed by the synthetic method) in a sequence – the situation unfolds as follows: the process begins *synthetically* with the A *Critique*, continues *analytically* with the *Prolegomena* (respecting the order *synthetic → analytic*), incorporates the results of the *Prolegomena* into the B *Critique* (following the principle that "in synthesis, one starts from the results of the method of analysis" – see Polya, section I of this essay). Since the *B Critique* necessarily presupposes the incorporation of the "results" of the *Prolegomena* and thus relies on the prior deductions from the *Prolegomena*, *the analytic* (*Prolegomena*) → *synthetic* (*B Critique*) order is also respected. Finally, the B *Critique* as a standalone work adopts Newton's mathematical method, transcendentally projected into the "generally" *synthetic* method of the B *Critique* as an experiment of pure reason, where both methods are combined at the fundamental levels of Kant's program[89].

We have seen that Newton also prioritizes the analytic method, but it should be noted that the sense in which Kant adopts this method, both in the *Prize Essay* and, more importantly, in the B *Critique*, is fundamentally synthetic. Ultimately, in the B *Critique*, Kant unfolds his entire program in the form of an experiment of pure reason following the "generally synthetic" procedure, where the two strands that underlie it – one *synthetic* and the other *analytic* – are evident (these procedures can be identified both in the deductions from the Aesthetic and from the Analytic)[90]. Perhaps most significant, as outlined in the first section, is that the results of the *Prolegomena* (achieved by Kant analytically) are incorporated into the B *Critique*, initially in the Introduction[91] and later as the analytic-transcendental strand of the experiment[92].

[89] For further, more detailed arguments regarding this order, see my study mentioned before: M.A. Drăghici, "Kant on Metaphysics as Science", *Revue roumaine de philosophie*, no. 2/2022, pp. 297–314.

[90] *Ibidem.*

[91] B 1 – B 24 (in this respect, see *ibidem*).

[92] In B 41, Kant introduces the "argument from geometry", which has its source in what he developed in the *Prolegomena*, where he began with the inquiry about the *possibility* of synthetic a priori knowledge in pure mathematics, leading to the possibility of geometry by analytically demonstrating its possibility from the pure intuition of space.

Thus, in a way, with some reservations, we can also agree with Loparic, who argued that the analytic method must precede the synthetic method – the results incorporated underscore the necessity of the analytic's role, characteristic of the approach in the *Prolegomena*, without which the experiment ("generally synthetic") in the B *Critique* would not have been possible.

These procedures are indeed of mathematical origin, adopted by Kant into his critical program in the manner described above only after the development of the doctrine of pure a priori intuition and the procedure of mathematical construction within pure intuition. It is only in the B edition that the complex adaptation and reformulation of these methods – *via* the experiment of sciences (Newton's, the physicists', and the chemists') – are fully transfigured to the domain of the *a priori* and the conditions of possibility for pure reason.

What I hope to have showed, however, is that, surprisingly, Kant had this model in mind – the synthetic (*a priori*) method of mathematical origin – since his youth. This model is comparable to Newton's (though interpretated since the *Prize Essay*), and Kant has been considered it ever since as akin to that of "true metaphysics". The rigorous distinction between the method of mathematics and that of philosophy remains important because, in mathematics, we have what philosophy does not: construction in pure a priori intuition. However, this does not weaken Kant's theory; on the contrary, it strengthens it, for in pure reason we have the deductions (metaphysical and transcendental) and the two methods (synthetic and analytic) from mathematics transcendentally reprojected. These methods are reprojected as integrated into the experiment of pure reason, ensuring even the *analogous* role of "construction in pure intuition", specific to mathematics.

The presence in the *Critique* (B) of elements related to pure natural science or the science of geometry can be understood (as an analytic-transcendental moment) in the context of outlining the "transcendental experiment of pure reason". This speaks – through a comparison with the experiments of physicists and chemists – about the possibility of determining, *in general* (initially through the synthetic-transcendental aspect), the relation of an *a priori* structure to objects of experience *via* pure concepts understood as conditions of the possibility of experience itself, including their application to the empirical domain. Ultimately, the analytic-transcendental aspect becomes relevant, as it contains the "proof of the realization of the categories". The success of the experiment, however, depends on the "identity of the results" – as Kant himself expressed it – on the concordance between the synthetic-transcendental and analytic-transcendental aspects. This concordance also implies the deduction from the *Prolegomena* (including its success in demonstrating that *the same fundamental elements are reached as in the synthetic approach of the A edition*). To the extent (and only to the extent) that their results coincide – that is, that they reach the same *a priori* principles of experience (the forms of sensibility and understanding: space, time, and the categories under the concept of the form of experience) – the hypothesis of the experiment is confirmed. Furthermore, this "coincidence" is precisely what I interpreted in the first part in connection with the definitions of the two methods in Pappus's account, when analyzing the coherence of the claim that "the result of the method of analysis is the result of synthesis".

IV. SOME FINAL COMMENTS ON HEURISTICS – METHOD RELATIONSHIP

In this last section I attempt to show that both Loparic and I have demonstrated the heuristic character of mathematics in Kantian transcendental philosophy, which consists in the influence of the two methods of geometers (Loparic's view), or, as observed in the B edition of the *Critique*, of these methods and "of the pure sciences in general" (my view). My view (as presented in sections II and III of this paper) involves the transformation and adaptation of the two methods, analytic and synthetic, within the experiment of pure reason in order to successfully verify the fundamental hypothesis: that we relate to phenomena as objects and not to things in themselves. If we complicate matters a bit, it becomes very difficult, or almost impossible, to assert that *this* or *that* was the "heuristics" of mathematics in Kantian transcendental philosophy. The two perspectives (Loparic's and mine) attest to this, although they are closely related.

As I will summarize below, Loparic interpreted Kant's *Critique* as a genuine transcendental semantics, with a strong connection to how the relationship to objects is understood through the reconfiguration of Kant's concept of the "significance" of the categories. This interpretation differs from my own, for I try to show that transcendental philosophy in the B *Critique* is a second-order theory, a framework theory, where the two mathematical methods are transcendentally redesigned within the "experiment of pure reason".

As a connection with the first section of my essay, I will begin with a brief commentary on the relationship between heuristics and methodology in some of Kant's references to method (in the *Methodology*) and to the term "heuristics" (throughout *Critique* B, which I consider to represent the mature theory of Kant's program). Specifically, I aim to address what in the B *Critique* is accounted for as the *heuristics* of mathematical methods in philosophy and/or in the theory of the *Critique*, and what pertains to "methods" in the *Methodology*: what are the methods that Kant validated in the *Methodology* for mathematics and philosophy, and which are the methods that *Kant himself employed* in constructing his own transcendental program, respectively those methods concerning transcendental philosophy[93]?

Kant distinguishes in A 713/B 741 between the mathematical method and the philosophical method (as *dogmatic*). The former characterizes mathematical

[93] If the transcendental philosophy constitutes the very program of the *Critique*, then the two coincide.

knowledge (through the construction of objects in pure intuition), while the latter characterizes philosophical knowledge ("from concepts"). Kant reiterates what he had maintained since his earlier writings, namely, that "mathematics and philosophy are two entirely different things, [...], thus that the procedure of the one can never be imitated by that of the other."[94] Regarding *heuristics*, the term is used five times in the *Critique*: three times adjectivally, in an associative synonymy with ideas, principles, or regulative concepts of reason, as in the expressions "heuristic and **regulative**" (principles), "heuristic principles," and the ideas of reason "as heuristic fictions"[95]; once in identifying the idea as a heuristic rather than an ostensive concept ("the idea is only a heuristic and not an ostensive concept"[96]); and once in connection with ensuring the algebraic method's protection against errors, from which heuristics are excluded ("without even regarding the heuristic, [algebraists' procedure] secures all inferences against mistakes by placing each of them before one's eyes"[97]).

It is clear that Immanuel Kant does not conceive of heuristics as the principal method for solving problems. He distinguishes it from the algebraists' method or the ostensive sense when referring to the idea as a "heuristic concept", considering ideas as "heuristic fictions". We could argue that, in this context, Kant embraces the "weak sense of heuristics"[98] (see the first section of this paper). Regarding methods, the distinction from Kant's earlier works is preserved, with the addition of "construction in pure intuition" for the mathematical method, introduced with the onset of the critical period (the *Inaugural Dissertation*, 1770), as well as the framework of the "conditions of possibility" and the "necessity of deductions" as part of the program of the *Critique*.

However, it is crucial to highlight that throughout the Methodology, Kant does not address the *method of transcendental philosophy* (that of the *Critique*) except in a "negative" sense: "Of the special method of a transcendental philosophy, however, nothing can here be said, since we are concerned only with a critique of the circumstances of our faculty – whether we can build at all, and how high we can carry our building with the materials that we have (the pure *a priori* concepts)."[99] It is therefore clear that, when Kant speaks of the method of philosophy, he is not

[94] A 727/B 755.

[95] A 617/B 645; A 664/B 692; A 771/B 799.

[96] A 671/B 699.

[97] A 735/B 763.

[98] It is worth noting here that, following Rescher, Loparic sees Kant's philosophy in general as a method for posing and solving human problems, with the *Critique* and *Prolegomena* representing, in this sense, the expression of a "tradition" that originated with the ancient mathematicians, whose goal was the "solution of problems" (in mathematics and algebra). In this tradition, brought to light by Vaihinger, and sharing a particular type of rationalism, Loparic includes Descartes, Leibniz, and, more recently, Popper and Kuhn, as well as the results of the theory of computability and solvability, or perspectives from cognitive psychology and studies in artificial intelligence. In this respect, see Z. Loparic, "On the Unavoidable Tasks of Pure Reason", pp. 194, 196.

[99] Imm. Kant, *Critique*, A 738/B 766.

referring to transcendental philosophy, nor, consequently, to the method of the *Critique*. This supports the argument in the second section of my paper, namely, that it remains to be investigated what is the implicit or explicit influence of mathematical methods on the methodological arsenal employed in constructing the program of the *Critique*. Similarly, the question of whether the mathematical method can be imitated in philosophy is still open, as outlined in the conclusions of the second section of this essay.

The above considerations (including the second section) and, additionally, Kant's specification at B 766 authorize us to assert that the two mathematical methods, together with Newton's (mathematical) method, are fundamental to the construction of the programe in the *Critique* (particularly in B) and to transcendental philosophy in general. However, the *role* of these methods is considered differently by Loparic and me, despite numerous points of convergence. One could argue that different *heuristics* concerning the problem of transcendental philosophy (i.e., the issue of the coherence/interpretation of the programe of the *Critique* and of Kant's theory as such) lead to different results (Loparic's interpretation *vs.* my interpretation of the *Critique*). The inflection point lies in our differing understandings of these influences in addressing the problem of transcendental philosophy. This raises the question: should these "methods" be regarded as *heuristics* or as actual *methods*? In other words, do they pertain to heuristics or to methodology? The first issue can be addressed below by briefly presenting Loparic's results, which interpreted the program of the *Critique* as a transcendental semantics, while the answer to the second will provide the opportunity for the analysis in the final section of this essay. Anticipating the synthesis of these two ways of problematizing, I would now suggest that, ultimately, the outcomes (both Kant's and our inquiries') *constitute knowledge*, and what ultimately differentiates this knowledge, at its core, is the differing consideration of these methods within the Kantian system.

I will revisit below the difference between how Loparic considered the order and significance of the two methods in the Kantian transcendental philosophy and how I have approached them, focusing initially on Loparic's perspective (my own viewpoint being elaborated more extensively in sections II and III).

Beyond the common elements highlighted at the beginning of this text, it also worth paying attention to the differences. More specifically, it should be noted that, due to his *methodological* perspective (different than my own) on approaching the B *Critique*[100], Loparic does not place the two methods within the historical-systematic context of the *B edition*. Therefore, he endorses an interpretation of the *order* of the two methods based on solely a few elements from the Kantian texts, relying primarily on the passage in the note at B 394 (from the Transcendental

[100] See M.A. Drăghici, "Some Methodological Aspects towards an Interpretation of Kant's *Critique of Pure Reason*", in *Revue roumaine de philosophie*, vol. 56, 2/2012, pp. 299–312.

Dialectic). This is the sole instance where Kant appears to give precedence in an order to the analytic method over the synthetic one[101].

I will revisit here two of the clearest "places" in Kant where the priority of the synthetic method for the systematic and demonstrative development of the transcendental philosophy is obvious: the fragment from the Preface and the longer and more explicit fragment from *Prolegomena* §4, which consider both the method of the A *Critique* (synthetic) and the method of the *Prolegomena* (analytic) – in this order.

> „Hier ist nun ein solcher Plan nach vollendetem Werke, der nunmehr nach analytischer Methode angelegt sein darf, da das Werk selbst durchaus nach synthetischer Lehrart abgefaßt sein mußte, damit die Wissenschaft alle ihre Artikulationen, als den Gliederbau eines ganzen besonderen Erkenntnisvermögens, in seiner natürlichen *Verbindung vor Augen stelle*."[102] [my emphasys].

As I have already pointed out in the second section, due to the defective translation from German into English, the meaning and significance of the expression "*before my own eyes*" have been lost. This expression is exactly the same as the one Kant used admiringly when discussing the synthetic method of geometry in his early works. By restoring this expression from the original text, I wish to highlight its significance in the context of the method of synthesis. Below is the passage from §4:

> In the *Critique of Pure Reason* I worked on this question *synthetically*, namely by inquiring within pure reason itself, and seeking to determine within this source both the elements and the laws of its pure use, according to principles. This work is difficult and requires a resolute reader to think himself little by little into a system that takes no foundation as given except reason itself, and that therefore tries to develop cognition out of its original seeds without relying on any fact whatever. *Prolegomena* should by contrast be preparatory exercises; they ought more to indicate what needs to be done in order to bring a science into existence if possible, than to present the science itself. They must therefore rely on something already known to be dependable, from which we can go forward with confidence and ascend t the sources, which are not yet known, and whose discovery not onlywill explain what is

[101] "In a systematic representation of those ideas, the suggested order, which is a **synthetic** one, would be the most appropriate; but in working through them, which must necessarily be done first, the **analytic** order, which inverts this one, is more suitable to the end of completing our great project, proceeding from what experience makes immediately available to us from the **doctrine of the soul**, to the **doctrine of the world** and from there all the way to the cognition of **God**." (*Critique*, B 394 n.).

[102] „Here then is such a *plan* subsequent to the completed work, which now can be laid out according to the *analytic method*, whereas the *work* itself absolutely had to be composed according to the *synthetic method*, so that the science might present all of its articulations, as the structural organization of a quite peculiar faculty of cognition, in their natural connection." (*Prolegomena*, p. 13 [4:264]).

> known already, but will also exhibit an area with many cognitions that all arise from these same sources. The methodological procedure of prolegomena, and especially of those that are to prepare for a future metaphysics, will therefore be *analytic*.[103] [4:275]

It is clear from the famous fragment above that the synthetic method precedes the analytic method in demonstrative and systematic order (see also the second section of this essay). On the other hand, unlike me, although he acknowledges that Newton's method was present in Kant's references regarding what the metaphysical method should be, Loparic argues that, given that Newton's method is based on mathematical methods, the latter – analytic and synthetic (in this order) – remain relevant. This order aligns with what Loparic believed should represent the sequence of these methods in mathematics.

With respect to how Kant effectively adopted these methods in his theoretical program, as well as to their definition, place, and role in Kant's philosophy, Loparic – unlike me – considers them to have remained ambiguous[104]. Moreover, his interpretative reconstruction did not have as one of its fundamental aims the search for coherence in these, admittedly sometimes unclear, elements *in Kant*. Loparic's interpretative evaluation here is guided by the (heuristic) interest in resolving this problem according to *his semantic perspective on Kantian transcendental philosophy*, which I will briefly address below. Even if the priority of the analytic method supports Loparic's interpretation of the transcendental philosophy as a transcendental semantics, Kant's references to the priority of the synthetic method over the analytic one cannot be minimized.

For the interpretation of transcendental philosophy as a "transcendental semantics", Loparic needed to prioritize the level of "meaning" of the categories, their realization, and this level is related to their connection to the empirical. In *A Semântica Transcendental de Kant*, Loparic states that the Kantian solution to the fundamental problem of transcendental philosophy is possible through the method of analysis and synthesis. This can be formulated as follows: How are synthetic propositions possible? In its most general form, Kant's central problem does not refer only to synthetic a priori propositions, as is often believed, but also to synthetic a posteriori propositions. In fact, the question of the possibility of determining the objects of experience also includes the question of the possibility of a posteriori propositions[105]. This aspect, emphasized by Loparic, is related to another point he reveals, namely that one of the rules of the method of "true metaphysics" (which I also mentioned in section II) would involve the "principles in constructing explanations for *other*, more complicated empirical events"[106].

[103] Imm. Kant, *Prolegomena*, pp. 25–26.
[104] Z. Loparic, "Kant's Philosophical Method" (I), pp. 475–476.
[105] Z. Loparic, *A semântica transcendental de Kant*, third edition, p. 49 (my translation).
[106] Z. Loparic, "Kant's Philosophical Method" (I), p. 472.

Loparic makes a useful observation regarding the heuristic aspect of the definitions of empirical concepts, noting that they function as “valuable instructions for new research”; even though they are essentially incomplete, empirical concepts can serve as the foundation of knowledge of the objects to which they refer. Then, the involvement of the analytic and synthetic methods in the metaphysical and transcendental deductions is decisive, which I also consider. This involvement leads Loparic to consider that the transcendental exposition, which unfolds under the combined method, is responsible for discovering the categories as necessary conditions for this semantic fact[107]. Thus, Kant’s theory of categories would be part of his theory of truth. Categories belong to the metalanguage in which Kant studies the truth conditions of the logical forms of judgments. Categories are introduced to conceptually (discursively) express determinations that objects must possess in order for judgments with determined a priori forms to be true in their case. In other words, categories are semantic and not syntactical concepts employed in order to characterize in an abstract way the truth conditions of synthetic a priori judgments[108]. Thus, the transcendental logic becomes a *semantic* logic, and Loparic paraphrases the famous Kantian phrase as follows: “The proud name of ontology has given place to a modest theory of intuitive reference and truth, that is, to intuitive (constructive) *a priori* semantics.”[109]

Loparic concluded that, similarly to contemporary analytic philosophy, Kant’s epistemological questions are actually semantic; they are independent of, and precede, those of epistemology. Transcendental logic is an a priori science that deals only with the laws of understanding and reason “as far as they refer to a priori objects” (B 81). Transcendental logic indeed progresses entirely a priori, without consulting experience. It uses the so-called “transcendental” knowledge by which we know “what and how certain representations”, including concepts, “are applied entirely a priori or are possible” (B 80). For this reason, transcendental logic can be interpreted as an a priori theory of the meaning of concepts and the truth of judgments within the domain of interpretation that encompasses natural phenomena accessible to intuition. In contemporary jargon, it is a constructivist a priori or transcendental semantics. In this respect, Loparic argues, it is easy to understand why transcendental logic includes an “aesthetics”: because a problem is solvable if, in its formulation, we use only predicates that refer to objects that can be given to us, and the theory of the “giving” of objects of knowledge is an essential part of the theory of determined predicates. For Kant, an object is “givable” – and, in this sense, possible – if it can be experienced, i.e., if it can be given in external or internal sensible intuition. In Loparic’s interpretation, the theory Kant calls “transcendental aesthetics” provides us with the domain of interpretation for synthetic a priori theoretical judgments: the sphere of possible experience.

[107] Z. Loparic, “Kant’s Philosophical Method” (II), p. 364.
[108] *Ibidem*, p. 365.
[109] *Ibidem*, p. 374.

Loparic describes a procedure of "sensification" (*Versinnlichung*) that I not revisit here *in extenso,* starting with the construction of figures and magnitudes in pure intuitions and ending with the application of sensible concepts to empirical objects. What we do retain, however, here is his emphasis that this procedure was practiced by mathematicians since ancient Greece. Kant would have completed his transcendental semantics with a theory of *a priori* methods for solving problems. His methodology would consist of a theory of proofs, to which he adds a program for a priori scientific research that provides scientists with (i) procedures for establishing rational fictions useful in the search for and organization of empirical facts, and (ii) procedures for finding empirical explanations (explanatory hypotheses) for those facts. Kant's method of proof is, essentially, the combined method of analysis and synthesis.

However, I have shown that it is more than the mathematical/geometrical method, because there is an evolution of Kant's theoretical program, even from the pre-critical period; and the relationship with mathematics and the two methods has a tradition in Kant that should not be overlooked. On the contrary, it should be taken into consideration. In this sense, I revisited the issue of "imitating" the mathematical method in philosophy (metaphysics), and through my analyses (based on some results from my previous works), I have attempted to trace their development up to the B edition of the *Critique of Pure Reason*, where we find Kant's mature program, and where the general problem of transcendental philosophy is addressed within the framework of the "experiment of pure reason". I have shown that here Kant speaks about his experiment as analogous to that of physicists and chemists, where both mathematical methods (synthetic and analytic – in this order) are implicitly involved.

As announced earlier, the final commentary of my paper will include some references to the perspective of Timothy Williamson[110], the analytic philosopher who in his recent researches directly addresses the problem of heuristics in knowledge. Williamson adopts the "weak" sense of heuristics, as I defined it in the first section. Under this understanding, the concept of "heuristics" should be considered not only from the perspective of cognitive psychology, as heuristics represent empirical rules for reasoning, where an educated approach reduces or limits the search for solutions in difficult and poorly understood domains. Unlike formal structures such as algorithms, heuristics do not guarantee optimal or even feasible solutions, and they are often used without any theoretical guarantee. This sense of heuristics also differentiates it from probabilistic, statistical, or rationalist reasoning, according to which people use rational and systematic methods to solve problems and generally seek optimal outcomes.

According to Williamson, a heuristics is a *rule of thumb* for solving problems of some type. The application of the rule may be automatic or deliberate;

[110] Timothy Williamson, "Heuristics and Paradoxes", in *Revue roumaine de philosophie,* 67, no. 2, 2023, pp. 313–332.

it may be conscious, unconscious, or somewhere in between. Even if it involves conscious activity, one may or may not know what rule one is applying, and one may or may not think of it as a heuristics. Even on reflection, it may not be obvious to us when we are using a heuristics, still less what heuristics it is[111]. The function of a heuristics is to provide a way of solving problems of a given type, that is fast, easy, efficient, and reliable enough to be useful. The way must be feasible in real time. It can be reliable enough without being perfectly reliable. Reliability here is equated with the probability that the way provides a correct solution, where the standard of correctness is built into the specification of the problem[112].

Although Williamson uses a broad concept of heuristics, he distinguishes between ordinary, pre-reflective human heuristics and more refined, philosophical, reflective heuristics. Williamson's research focuses on clarifying the theoretical understanding of the potential relevance of specific heuristics for philosophy. Heuristics is a subject often approached from certain perspectives as a potential source of errors that must be corrected and continuously monitored. For instance, in Williamson's view, some "philosophical paradoxes are the result of our reliance on efficient but fallible humanly universal heuristics. This is illustrated in relation to paradoxes concerning vagueness and conditionals."[113]

The general characteristics employed in these approaches would be as follows: Heuristics do not rely on a solid or fully demonstrated theoretical foundation. They provide practical and efficient solutions, often based on experience or intuition, without guaranteeing absolute correctness. Heuristics have limited applicability to specific contexts and are not necessarily universal. This process is often implicit or tacit, relying on informal judgments, and tends to prioritize optimization, speed, and the use of shortcuts to reach that "good enough" solution in a short amount of time. This endeavor tends to complicate the relationship between heuristics and methodology. However, the same heuristics cannot be denied the quality of being a "source of inspiration"; what is its status in relation to methodology? Methodology, on the other hand, is based on a solid theoretical foundation and is rationally and logically justified. It has a universal character, being applicable across different contexts and providing solutions that are verifiable and replicable. Methodology is explicit and clearly articulated, with each step documented and supported by clear arguments. It follows a rigorous process, without shortcuts, and prioritizes accuracy and the validity of solutions, even if this means a longer process.

Some of Williamson's findings are that heuristics apply equally well to both precise and vague cases, but often heuristics allow us to make our *implicit* bias or prejudice explicit in a conditional or in a generic generalization. The ability to filter and select a heuristics as better or worse is therefore limited. For the utility of a heuristics depends on its unrestricted simplicity. This would also lead to the

[111] *Ibidem*, p. 313.
[112] *Ibidem*, p. 329.
[113] *Ibidem*.

necessity of treating heuristics locally, not globally[114]: an unlimited universal generalization will inevitably be false and, therefore, without a basis for knowledge. However, Williamson's optimism is cautious: we should be wary of drawing pessimistic methodological conclusions for philosophy from our trust in fallible heuristics. Heuristics are not specific to philosophy; they support much of our thinking in general. Since our trust in them does not justify generic skepticism, assuming it justifies specific philosophical skepticism would be arbitrary[115].

Our concluding comment aims to focus on the connection between heuristics and methodology, which Williamson makes at the end of his research. He believes we should ask what improvements could be made to our current philosophical methodology to make it less vulnerable to the illusions induced by heuristics. Even though sometimes when we correct the results of heuristics, we may rely on other heuristics or even on other applications of the same heuristics, Williamson argues that methodological improvements are feasible and will challenge some currently fashionable ideas. However, the nature of knowledge is of this type, and heuristics are merely a structural manifestation of this situation, a 'disposition', as Kant would say, something important and relevant nonetheless. A possible methodological approach to heuristics, aimed at avoiding errors and paradoxes as well as validating a heuristics, can be summarized in Williamson's maxim: "the degree of reliability of a heuristics can be identified with the objective probability of true outcomes conditioned by true inputs"[116]. More specifically, if the heuristics is inferential, with inputs resembling premises, what matters is maintaining the truth from inputs to output, rather than just the truth of the output itself. The degree of reliability can be identified with the relative frequency of true results given true inputs[117]. And if the result of the heuristics is an estimate rather than a judgment, it can be evaluated on a graded scale of accuracy, rather than on the binary distinction between truth and falsity.

To conclude, according to Williamson, all these reliability standards can be relativized to the conditions under which heuristics was applied. What Williamson emphasizes is that, despite all these complications, reliability is still defined in terms of a standard of truth or accuracy given completely independently of the heuristics itself. More specifically, no role has been assigned to heuristics in determining the content of the judgments or estimates it produces[118].

I will now discuss this relationship between heuristics and methodology, considering the two previous sections of our essay as parts of an experiment in which we ultimately tested this relationship through the lens of the two meanings attributed to heuristics in the first section (the 'strong' and the 'weak' sense).

[114] *Ibidem*.
[115] *Ibidem*, p. 330.
[116] *Ibidem*, pp. 330–331.
[117] *Ibidem*.
[118] *Ibidem*.

In this experiment (of this essay), however, I have tried to show that these interpretations (Loparic's and mine), although different, start from almost the same premises (which generally coincide in terms of evaluating their truth – that the two methods have a mathematical origin, that they are interpreted in a similar way, and that they had major relevance in Kant), and have led to different results based on different *heuristics* (ours) regarding *the solution to the system's problems or the difficulties of Kant's transcendental philosophy* (the idea of 'making Kant more coherent and consistent with himself'), starting from the place, role, and relevance (*heuristics*) of mathematical methods in *Kant's solution* to the problem of transcendental philosophy. In the already published/written texts concerning Kant's transcendental philosophy, my reference was *implicit* to the *heuristics* of mathematical methods in Kant's writings, but *explicit* regarding the nature and structure of Kant's first *Critique* from the perspective of solving the transcendental problem through *methods that had an influence on Kant's theoretical program*.

However, with this essay, I have explicitly raised the issue by analyzing these aspects: I have called this *heuristic* influence (although in previous works I only spoke about *influence*). In our works, we have largely used the Kantian arsenal that underpinned the development of the transcendental theory as if we were playing the role of those whom Kant often said could present or continue his system. Keeping only the idea from what has been said, both Loparic and I have often suggested or even stated that 'Kant would agree with this' or that 'Kant actually achieved such and such a thing' but could not say it quite this way at that time for obvious reasons, etc. All of this contributes to the formation of two different perspectives (his and mine) on how to use, in one way or another, the synthetic and analytic methods (in a kind of strictly minimal complement to what Kant has already achieved, *implicitly*, regarding these methods) in the (interpretative, partially reconstructive) construction of the Kantian theoretical program.

Yet the way in which the elements in Kant are evaluated, reconstructed, and reinterpreted underlies *a knowledge perspective on the Kantian program*, and implicitly, *on the problems addressed by Kant and on the validity of the solutions provided (by Kant and by the two perspectives as such*). Therefore, we could discern three different levels of the heuristics of mathematics in Kant: the level of the more or less implicit one in Kant's early works and his critical period, where we have a heuristics in the form of reliance on mathematical methods in solving the problem of metaphysics/transcendental philosophy; the level at which heuristics is assumed as an *influence* in our approaches to Kant's program itself (unlike me, Loparic makes some explicit references to heuristics); and the *meta*level of this essay, where heuristics is consciously addressed and analyzed as an *explicit heuristics*.

Generally speaking, an author may argue that his/her own perspective is more effective than others. Here, as I mentioned, it would be a *knowledge perspective* on the *Kantian program* and, implicitly, *on the problems addressed by Kant and the validity of the solutions provided to these problems* (*by Kant and by the* *perspective as such*). However, this perspective, like the other one, has done nothing but start from what both assumed as similar: from the *heuristics* of mathematics in Kant's task of solving the problem of metaphysics/transcendental philosophy. Yet the two perspectives have ultimately claimed that *they* explain how the influence (heuristics) of the two methods should have been exercised and interpreted. Moreover, these perspectives argue, at their limits, that they are more effective than Kant's explanations regarding the nature and purpose of his own program, through a greater underlying coherence, with a superior validity granted by certain accommodations of the meanings and structures of the theory as such, and with an adaptive and performant advantage in respect with the new challenges of knowledge after Kant, etc.

Indubitably, what I listed above regarding these two perspectives (but this model can be extrapolated to all more or less similar approaches) is indeed a certain type of knowledge (philosophical, let's say). In fact, what I want to argue here is that 'heuristics simply produces knowledge'.

One more element should be emphasized here: the need for further investigation of the distinction between heuristics and methodology, so radically outlined in analytic philosophy. The two methods (initially heuristics in the weak sense) have become a structural and fundamental part of the Kantian transcendental method in the exemplary form of the transcendental deductions circumscribed by the broader framework of the 'experiment of pure reason' – this is my standpoint. In this sense, one of the natural research openings would be to investigate the heuristics of the Kantian critical program on mathematical intuitionism – that is, to what extent the doctrine of construction in pure a priori intuition influenced the mathematical intuitionism initiated by Brouwer in the middle of the last century. As it is well-known, Brower himself acknowledged these influences.

Relying to some extent on the previous two sections, in this final section I have shown that, although many elements of our interpretations (mine and Loparic's) regarding the two mathematical methods (analysis and synthesis) in relation to the problem of transcendental philosophy and the transcendental method are similar, the perspectives are different – these statements can be found in our *already* published works; that these different perspectives are based on heuristics regarding the two methods in Kant and the Kantian theoretical program; that the perspective from analytic philosophy (Timothy Williamson) on the relationship heuristics-methodology-knowledge requires a correction; that, in the end, 'heuristics simply means knowledge' – but what about method(s) and methodology?