

TWO VALIDITY PROBLEMS IN FIRST-ORDER PREDICATE  
CALCULUS THAT A NON-AXIOMATIC LOGIC  
CAN OVERCOME

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**Abstract:** In this paper, I analyze two inference tasks with a disjunctive premise. People accept or reject their conclusions in a way different from First-Order Predicate Calculus. I review the inferences based on the framework of Non-Axiomatic Logic. Several components of the latter logic allow coming to the same conclusions as human beings. Two important concepts of the logic are that of intensional intersection, which enables to express disjunctions, and that of extensional difference, which enables to indicate when a disjunction is exclusive.

**Keywords:** disjunction, extensional difference, inference, intensional intersection, Non-Axiomatic Logic.

INTRODUCTION

It is well-known that First-Order Predicate Calculus cannot address all the responses people give in experimental reasoning tasks.<sup>1</sup> There are inconsistencies between what individuals deem as correct and what is valid in classical logic. Two representative examples are (1) and (2).

(1) “*Few people ate steak or sole; therefore, few people ate steak*”.<sup>2</sup>

(2) “*He had the anesthetic or felt pain, but not both; therefore, he had the anesthetic or felt pain, or both*”.<sup>3</sup>

In First-Order Predicate Calculus, (1) is not valid and (2) is valid. However, people tend to accept (1) and reject (2).<sup>4</sup> There is a cognitive proposal giving an

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<sup>1</sup> E.g. Isabel Orenes, Philip N. Johnson-Laird, “Logic, models, and paradoxical inferences”, *Mind & Language*, vol. 27, nr. 4, 2012, pp. 357–377.

<sup>2</sup> Philip N. Johnson-Laird, Ruth M. J. Byrne, Sangeet Khemlani, “Models of possibilities instead of logic as the basis of human reasoning”, *Minds & Machines*, vol. 34, nr. 19, 2024, p. 1; italics in text.

<sup>3</sup> *Loc. cit.*; italics in text.

account of this fact: the theory of mental models.<sup>5</sup> The aim of this paper is to show that there is also a non-axiomatic logic that can do that. The logic is Non-Axiomatic Logic (NAL).<sup>6</sup>

Unlike the theory of mental models, NAL is not intended to describe the way people think. It only tries to come to the same conclusions as people, even if that is made by means of different inferential processes.<sup>7</sup> So, it does not rival the theory of mental models. On the other hand, there is a computer system that relies on NAL. It is Non-Axiomatic Reasoning System (NARS).<sup>8</sup> This means that if I show that NAL can deal with (1) and (2), I will also show that NARS can do the same.

The general characteristics allowing NAL to come to the same conclusions as human beings in (1) and (2) will be described in the first section. In the second one, I will explain why NAL can derive the conclusion in (1) from those characteristics. An objection to my account will be considered and solved in the third section. In the last one, my solution to the objection will lead to the reasons why NAL can reject the conclusion in (2).

### KEY COMPONENTS IN NAL

NARS is a system with extensive machinery. It can be used to do many different tasks, including medical diagnosis<sup>9</sup> and substitution for recommender systems.<sup>10</sup> Besides, there are already papers addressing reasoning tasks challenging First-Order Predicate Calculus from NAL, or idealized versions of it.<sup>11</sup> However,

<sup>4</sup> *Ibidem*, pp. 1–22.

<sup>5</sup> See also, e.g., Philip N. Johnson-Laird, “Possibilities and human reasoning”, *Possibilities Studies & Society*, vol. 1, nr. 1–2, 2023, pp. 105–112; Philip N. Johnson-Laird, Ruth M. J. Byrne, Sangeet Khemlani, “Human verifications: Computable with truth tables outside logic”, *Proceedings of the National Academy of Sciences*, vol. 120, nr. 40, e2310488120, 2023, pp. 1–6; Philip N. Johnson-Laird, Marco Ragni, “Reasoning about possibilities: Modal logics, possible worlds, and mental models”, *Psychonomic Bulletin & Review*, vol. 32, nr. 1, 2025, pp. 52–79.

<sup>6</sup> A detailed description of it is to be found in Pei Wang, *Non-Axiomatic Logic: A Model of Intelligent Reasoning*, Singapore, World Scientific, 2013.

<sup>7</sup> P. Wang, *Non-Axiomatic Logic*.

<sup>8</sup> A detailed description of NARS is to be found in Pei Wang, *Rigid Flexibility: The Logic of Intelligence*, Dordrecht, Springer, 2006.

<sup>9</sup> E.g., Pei Wang, Seemal Awan, “Reasoning in Non-Axiomatic Logic: A case study in medical diagnosis”, in Jürgen Schmidhuber, Kristinn R. Thórisson, Moshe Looks (eds.), *Artificial General Intelligence. AGI 2011. Lecture Notes in Computer Science, volume 6830*, Berlin, Heidelberg, Springer, 2011, pp. 297–302.

<sup>10</sup> Julia Fasick, *Non-Axiomatic Reasoning Systems as a substitute for Recommender Systems* [unpublished course project]. Department of Computer and Information Sciences, College of Science and Technology, Temple University, 2025. <https://cis.temple.edu/~wangp/5590-AGI/Projects-2025/FasickJ/Non-Axiomatic%20Reasoning%20Systems%20as%20a%20Substitute%20for%20Recommender%20Systems.pdf>, accessed: 21 November 2025.

<sup>11</sup> E.g., Miguel López-Astorga, “Computer Chrysippus’ conditional and extensional intersection”, *Theoria*, vol. 91, nr. 5, e70033, 2025, pp. 1–6.

due to space constraints, I will only describe the characteristics of NAL necessary to address (1) and (2).

Perhaps, the main copula in NAL is the ‘inheritance copula’: ‘ $\rightarrow$ ’.<sup>12</sup> That copula allows writing in the language of NARS, which is named ‘Narsese’<sup>13</sup>, statements such as (3).

$$(3) \quad \textit{Wolf} \rightarrow \textit{Gray}$$

*Wolf* is the subject of the statement, and *Gray* is its predicate. (3) provides that *Wolf* belongs to the extension of *Gray*. It also indicates that *Gray* belongs to the intension of *Wolf*.<sup>14</sup> Let  $W^E$ ,  $W^I$ ,  $G^E$ , and  $G^I$  be, respectively, the extension of *Wolf*, the intension of *Wolf*, the extension of *Gray*, and the intension of *Gray*. We can use set theory as a metalanguage<sup>15</sup> and say that (3) establishes that

$$\{W^E\} \subseteq \{G^E\} \text{ and} \\ \{G^I\} \subseteq \{W^I\}$$

This approach of extension and intension, which is not exactly the habitual one, is complemented in NAL by the ‘Assumption of Insufficient Knowledge and Resources’ (AIKR).<sup>16</sup> This assumption implies that the system works like us: with lack of information and time. Hence, it can never be sure that a statement is true. This is the reason why each statement in NAL has two truth values. The first one is  $f$  (‘frequency’), and “ $f = w^+/w$ ”,<sup>17</sup> where  $w^+$  represents ‘positive evidence’, and  $w$  stands for ‘available evidence’. The second one is  $c$  (‘confidence’), and “ $c = w/(w + k)$ ”,<sup>18</sup> where  $k$  is a constant. The value of  $k$  is generally 1. However, its value can increase if we want the system to require larger amounts of evidence to accept statements and conclusions.<sup>19</sup>

In Narsese, a statement such as (3) should be expressed as follows:<sup>20</sup>

$$(4) \quad \textit{Wolf} \rightarrow \textit{Gray} \langle f, c \rangle$$

<sup>12</sup> E.g., Pei Wang, “The role of copulas in reasoning. Technical Report #17”, *Temple AGI Team. Temple University*, 2023. <https://cis.temple.edu/tagit/publications/TAGIT-TR-17.pdf>, accessed: 21 November 2025.

<sup>13</sup> E.g., P. Wang, *Non-Axiomatic Logic*.

<sup>14</sup> E.g., P. Wang, “The role of copulas in reasoning”.

<sup>15</sup> E.g., P. Wang, *Non-Axiomatic Logic*.

<sup>16</sup> An analysis of AIKR is to be found in, e.g., Pei Wang, “The assumptions on knowledge and resources in models of rationality”, *International Journal of Machine Consciousness*, vol. 3, nr. 1, 2011, pp. 193–218.

<sup>17</sup> P. Wang, *Non-Axiomatic Logic*, p. 29 (Definition 3.3).

<sup>18</sup> *Loc. cit.*

<sup>19</sup> E.g., P. Wang, “The role of copulas in reasoning”.

<sup>20</sup> P. Wang, *Non-Axiomatic Logic*.

Let us suppose that the system has information on 15 wolves, and only 10 of them are gray. In that case,  $f = 10/15 = 0.67$ , and  $c = 15/16 = 0.94$  (for  $k = 1$ ). The correct way to write (4) in Narsese would be:

$$Wolf \rightarrow Gray \langle 0.67, 0.94 \rangle$$

If the system acquires as new information a statement without values for  $f$  and  $c$ , in principle, it will assign, respectively, 1 and 0.9 to them.<sup>21</sup>

To work in Narsese with proper nouns, or words referring to a particular person or object, ‘extensional sets’ are used.<sup>22</sup> An extensional set is a set with just one element. That element often has no extension, only intension. If  $W_I$  is a particular wolf, the extensional set containing only  $W_I$  can be expressed in Narsese as  $\{W_I\}$ . This allows creating statements such as (5).

$$(5) \quad \{W_I\} \rightarrow Wolf \langle f, c \rangle$$

NAL has many inference rules. In this paper, I only need two of them: the ‘deduction rule’ and the ‘choice rule’. The deduction rule enables to make inferences such as the following:<sup>23</sup>

(A)	$Wolf \rightarrow Gray \langle 0.67, 0.94 \rangle$	Premise
(B)	$\{W_I\} \rightarrow Wolf \langle 1, 0.9 \rangle$	Premise
(C)	$\{W_I\} \rightarrow Gray \langle 0.67, 0.57 \rangle$	From A & B

The truth values of the conclusion, that is, statement (C), are calculated in this way. Let the frequency of the first premise and the frequency of the second premise be, respectively,  $f_1$  and  $f_2$ . If  $f_3$  is the frequency of the conclusion,  $f_3 = “f_1 \times f_2”$ .<sup>24</sup> Let the confidence of the first premise and the confidence of the second premise be, respectively,  $c_1$  and  $c_2$ . If  $c_3$  is the confidence of the conclusion,  $c_3 = “f_1 \times c_1 \times f_2 \times c_2”$ .<sup>25</sup>

On the other hand, the choice rule can help select the best statement in different senses.<sup>26</sup> The sense that is interesting here is that of a question such as

$$Wolf \rightarrow Gray?$$

The principle of explosion does not apply in NAL. That means that the same statement can be in the system with different truth values at the same time. The

<sup>21</sup> *Loc. cit.*

<sup>22</sup> *Ibidem*, p. 84 (Definition 6.3).

<sup>23</sup> To see the process in detail, see, e.g., P. Wang, *Non-Axiomatic Logic*.

<sup>24</sup> P. Wang, *Non-Axiomatic Logic*, p. 61 (Table 4.7).

<sup>25</sup> *Loc. cit.*

<sup>26</sup> For an explanation, see, e.g., P. Wang, *Non-Axiomatic Logic*.

reason for this is that the system can come to the same statement following different paths.<sup>27</sup> For example, experience can give the system this statement:

$$Wolf \rightarrow Gray \langle 0.67, 0.94 \rangle$$

But, because of the use of its inference rules, the system can also derive a statement such as

$$Wolf \rightarrow Gray \langle 1, 0.81 \rangle$$

In circumstances such as this one, if both statements have at least a common evidential element, the choice rule will lead to select the statement with higher  $c$ .<sup>28</sup> In this case, the statement will be the first one, that is, that with  $c = 0.94$ .

An important connective in NAL is the ‘product connector’ (‘x’). It is important because it allows providing ‘relations’.<sup>29</sup> One of those relations can be, for instance, that existing between *Wolf* and *Meat*: *Wolves Eat Meat*. In Narsese, the latter relation could be written as (6).

$$(6) \quad (Wolf \times Meat) \rightarrow Eat \langle f, c \rangle$$

The problem with a statement such as (6) is that it makes it difficult to apply rules such as the deduction rule. One of the resources of NAL to solve this difficulty is the ‘extensional image connector’ (‘/’). This connector enables to transform statements such as (6) into statements such as (7).<sup>30</sup>

$$(7) \quad Wolf \rightarrow (/ Eat \diamond Meat) \langle f, c \rangle$$

Values  $f$  and  $c$  are the same in (6) and (7), and (7) can also be transformed into (6). Regarding ‘ $\diamond$ ’, it just indicates the place of *Wolf* in the premise, that is, in (6). ‘ $\diamond$ ’ is before *Meat* in (7) because *Wolf* is the left term in (6) (for a detailed account, see Wang, 2013).<sup>31</sup> Thus, NAL can make inferences such as the following:

(D)	$(Wolf \times Meat) \rightarrow Eat \langle 1, 0.9 \rangle$	Premise
(E)	$\{W_1\} \rightarrow Wolf \langle 1, 0.9 \rangle$	Premise
(F)	$Wolf \rightarrow (/ Eat \diamond Meat) \langle 1, 0.9 \rangle$	From D
(G)	$\{W_1\} \rightarrow (/ Eat \diamond Meat) \langle 1, 0.81 \rangle$	From E & F
(H)	$(\{W_1\} \times Meat) \rightarrow Eat \langle 1, 0.81 \rangle$	From G

<sup>27</sup> *Loc. cit.*

<sup>28</sup> *Loc. cit.*

<sup>29</sup> *Ibidem*, pp. 107–110 (especially, Definitions 8.1 and 8.3).

<sup>30</sup> *Ibidem*, pp. 112–113 (Definition 8.4).

<sup>31</sup> For a detailed account, see P. Wang, *Non-Axiomatic Logic*.

Disjunction exists in NAL. It is isomorphic to what in NAL is deemed as ‘intensional intersection’.<sup>32</sup> The symbol for intensional intersection in NAL is ‘ $\cup$ ’. It does not have the same meaning as in set theory. It allows building an intensional intersection between, for example, *Wolf* and *Dog*:  $Wolf \cup Dog$ . That intensional intersection expresses the common properties both *Wolf* and *Dog* have in their intensions. We can also use First-Order Predicate Calculus as a metalanguage<sup>33</sup> and claim that

$$\forall x \{[(Wolf \cup Dog) \rightarrow x] \text{ IFF } [(Wolf \rightarrow x) \wedge (Dog \rightarrow x)]\}$$

A similar concept that will be necessary for my account below is that of ‘extensional difference’.<sup>34</sup> Its symbol in NAL is ‘-’. An expression such as *Gray - Wolf* refers to all the gray animals and things that are not wolves. Using First-Order Predicate Calculus as a metalanguage again, we can say that the extensional difference between *Gray* and *Wolf* is as follows:

$$\forall x \{[x \rightarrow (Gray - Wolf)] \text{ IFF } [(x \rightarrow Gray) \wedge \neg(x \rightarrow Wolf)]\}$$

The two last components I need are related to the way NAL can deal with what the existential quantifier means in First-Order Predicate Calculus, and the manner NAL negates statements. NAL has dependent variables.<sup>35</sup> Beyond their exact role in the logic, in this paper, I will consider them as existential quantifiers. This is because they can express what existential quantifiers express in First-Order Predicate Calculus.<sup>36</sup> In Narsese, ‘x is a dependent variable’ is said in this way:

$$\#x()$$

Which will be deemed as equivalent to ‘ $\exists x$ ’ in First-Order Predicate Calculus here.

Lastly, it is easy to negate a statement in NAL. When the system negates a statement, it considers the pieces of negative evidence to be pieces of positive evidence, and the pieces of positive evidence to be pieces of negative evidence.<sup>37</sup> The negation of

$$Wolf \rightarrow Gray \langle 0.67, 0.94 \rangle$$

<sup>32</sup> For an explanation of the concept of intensional intersection and the isomorphism, see P. Wang, *Non-Axiomatic Logic*, p. 94 (Definition 7.7) and p. 122 (Table 9.1).

<sup>33</sup> P. Wang, *Non-Axiomatic Logic*.

<sup>34</sup> For an explanation, see P. Wang, *Non-Axiomatic Logic*, p. 97 (Definition 7.8).

<sup>35</sup> For an explanation, see P. Wang, *Non-Axiomatic Logic*, pp. 139–141 (Definition 10.2).

<sup>36</sup> They can do that at least as far as the aims of this paper are concerned; see P. Wang, *Non-Axiomatic Logic*.

<sup>37</sup> P. Wang, *Non-Axiomatic Logic*, p. 129 (Definition 9.7).

Is

$$Wolf \rightarrow Gray \langle 0.33, 0.94 \rangle$$

These components suffice to address (1) and (2) within NAL. My developments below will be only possible ways NAL can deal with (1) and (2). The machinery NAL has is very extensive. It has many resources to explain the habitual responses people give to tasks with the structures of (1) and (2). So, it can also come to those very responses by means of other paths.

### INFERENCE (1) WITHIN NAL

The first task to show that NAL can derive the conclusion in (1) from its premise is to translate both into Narsese. The premise ('few people ate steak or sole') can be (8).

$$(8) \quad [\#x() x (Steak \cup Sole)] \rightarrow Ate \langle 1, 0.9 \rangle$$

In (8), dependent variable '#x()' refers to what in the premise is indicated in natural language as 'few people'. (8) provides a relation between the dependent variable and 'steak or sole'. The relation is that the people represented by the dependent variable ate steak or sole. The link between *Steak* and *Sole* is made by means of an intensional intersection because, as said, disjunction is isomorphic to intensional intersection in NAL. The truth values are those that the system assigns when it has no information on them.

On the other hand, the translation of the conclusion ('few people ate steak') can be (9).

$$(9) \quad (\#x() x Steak) \rightarrow Ate \langle 1, 0.9 \rangle$$

Which indicates that the individuals stood for by the dependent variable ate steak.

The work to do is to derive (9) from (8). That is possible in NAL. By definition, we know that (10) is the case.

$$(10) \quad Steak \rightarrow (Steak \cup Sole) \langle 1, 0.9 \rangle$$

One might think that the value of  $c$  in (10) should be 1. Let  $ST^I$  and  $SL^I$  be, respectively, the intension of *Steak* and the intension of *Sole*. It is obvious that  $\{ST^I \cap SL^I\} \subseteq \{ST^I\}$  (here, ' $\cap$ ' and ' $\subseteq$ ' have the same meaning as in set theory, which I am using as a metalanguage again). Despite that, I have attributed to (10)

the value NAL gives to  $c$  when it is unknown. This point is debatable. In any case, it is not relevant in this paper, as it does not have an influence on my argumentation below.

Statement (11) can be derived from (8).

$$(11) \quad (Steak \cup Sole) \rightarrow (/ Ate \#x() \diamond) \langle 1, 0.9 \rangle$$

From (10) and (11), we obtain (12).

$$(12) \quad Steak \rightarrow (/ Ate \#x() \diamond) \langle 1, 0.81 \rangle$$

In the system (12) and (13) are equivalent.

$$(13) \quad (\#x() \times Steak) \rightarrow Ate \langle 1, 0.81 \rangle$$

Statements (9) and (13) are the same. The only difference is that  $c$  is lower in (13). However,  $c$  keeps being high enough in (13). So, we can say that the conclusion in (1) is derived.

#### A POSSIBLE OBJECTION: (1) WITH A DIFFERENT CONTENT

Let us think about an inference such as that in (14).

(14) They went to the beach or to the countryside yesterday. Therefore, they went to the beach yesterday.

In this case, people are likely to act in accordance with classical logic and reject the conclusion. They can interpret that the two options represented by the disjuncts ('to go to the beach yesterday' and 'to go to the countryside yesterday') could not happen at the same time, that is, that the disjunction is exclusive.<sup>38</sup> This could mean an objection against my account in the previous section. But this problem admits a simple solution.

If the disjunction is interpreted as exclusive, the information in Narsese the system must receive needs to include extensional differences. A way to capture the fact that the disjunction in (14) is exclusive can be to assume that the intersection between the extension of *Beach* and the extension of *Countryside* is empty (remember that extension and intension do not have in NAL their usual meanings). Let  $B^E$  and  $C^E$  be, respectively, the extension of *Beach* and the extension of *Countryside*. In the language of set theory, we assume that  $\{B^E \cap C^E\} = \emptyset$ .

Thus, statement (15) can represent the first disjunct in (14).

<sup>38</sup> The theory of mental models can deal with examples such as this one, too; see, e.g., P. N. Johnson-Laird, M. Ragni, "Reasoning about possibilities".

$$(15) \quad [\#x() \text{ x } (Beach - Countryside)] \rightarrow Went \langle 1, 0.9 \rangle$$

And (16) the second one.

$$(16) \quad [\#x() \text{ x } (Countryside - Beach)] \rightarrow Went \langle 1, 0.9 \rangle$$

Because if the interpretation is exclusive, only one of the disjuncts can be true, only one of the latter two statements, that is, (15) or (16), can be true. So, the assumption of each of them is a different path to check whether we can come to the conclusion in (14).

But these are not the only two possible paths. Given that the premise in (14) is a disjunction, the system can work as shown in the previous section with (1). The premise in (14) could be expressed in Narsese as follows:

$$(17) \quad [\#x() \text{ x } (Beach \cup Countryside)] \rightarrow Went \langle 1, 0.9 \rangle$$

The system knows that (18) holds.

$$(18) \quad Beach \rightarrow (Beach \cup Countryside) \langle 1, 0.9 \rangle$$

And (17) can be transformed into (19).

$$(19) \quad (Beach \cup Countryside) \rightarrow (/ Went \#x() \diamond) \langle 1, 0.9 \rangle$$

The deduction rule allows inferring (20) from (18) and (19).

$$(20) \quad Beach \rightarrow (/ Went \#x() \diamond) \langle 1, 0.81 \rangle$$

Therefore, we obtain (21).

$$(21) \quad (\#x() \text{ x } Beach) \rightarrow Went \langle 1, 0.81 \rangle$$

Which corresponds to the conclusion in (14).

If this were the only possible path in the system, (14) should be accepted. The confidence in (21) is high enough ( $c = 0.81$ ). However, as indicated above, there are at least two more paths.

The first one is to assume that the first disjunct in (14) is the case, that is, that (15) is the case. If (15) is the case, we can say that there is at a minimum an element  $a$  satisfying (15). In other words, we can say that (22) is true.

$$(22) \quad [\{a\} \text{ x } (Beach - Countryside)] \rightarrow Went \langle 1, 0.9 \rangle$$

If (22) is true, (23) is true as well.

$$(23) \quad [\{a\} \times (\text{Countryside} - \text{Beach})] \rightarrow \text{Went} \langle 0, 0.9 \rangle$$

And this also holds for any element  $n$  such that  $n \neq a$  and  $n$  satisfies (15).

In this path, the choice rule would lead to reject (21) because  $c$  is higher (0.9 versus 0.81) both in (22) and in (23). Nevertheless, note that because  $\{B^E \cap C^E\} = \emptyset$ ,  $[\{a\} \times (\text{Beach} - \text{Countryside})] \rightarrow \text{Went}$  implies  $(\{a\} \times \text{Beach}) \rightarrow \text{Went}$ . So, the result would be to accept the conclusion in (14) again, but with a higher confidence level. If (22) and (23) are the case, every  $n$  satisfying (15) was in the beach yesterday.

One might challenge this. The objection could be that to apply the choice rule, statements must have common elements in their ‘evidential bases’.<sup>39</sup> However, there is at least a common element in the evidential bases: the premise in (14). (15) and (17) derive from the premise in (14). This applies to my similar arguments below, too.

This is not the last path. The third path is to consider (16) to be the case. If (16) is true, there is at least an element  $a$  satisfying (16). Hence, (24) is also true.

$$(24) \quad [\{a\} \times (\text{Countryside} - \text{Beach})] \rightarrow \text{Went} \langle 1, 0.9 \rangle$$

If (24) is true, (25) holds as well.

$$(25) \quad [\{a\} \times (\text{Beach} - \text{Countryside})] \rightarrow \text{Went} \langle 0, 0.9 \rangle$$

And this is correct for any element  $n$  such that  $n \neq a$  and  $n$  satisfies (16).

Statements (24) and (25) establish the opposite of what (22) and (23) show. Hence, they lead to reject the conclusion in (14). The choice rule cannot help the system decide. (22), (23), (24), and (25) all have the same confidence value ( $c = 0.9$ ). This explains why the conclusion in (14) is not accepted as true for sure if the disjunction is interpreted as exclusive. It can be true: there is a path leading to it with  $c = 0.9$ . But it can also be false: there is another path rejecting that very conclusion with  $c = 0.9$  as well. This allows understanding the case of (2).

## INFERENCE (2) WITHIN NAL

The premise in (2), that is, ‘he had the anesthetic or felt pain, but not both’, requires these three statements in Narsese:

$$(26) \quad [\{He\} \times (\text{Anesthetic} \cup \text{Pain})] \rightarrow (\text{Had} \cup \text{Felt}) \langle 1, 0.9 \rangle$$

$$(27) \quad [\{He\} \times (\text{Anesthetic} - \text{Pain})] \rightarrow (\text{Had} \cup \text{Felt}) \langle 1, 0.9 \rangle$$

<sup>39</sup> E.g., P. Wang, *Non-Axiomatic Logic*.

$$(28) \quad [ \{He\} \times (Pain - Anesthetic) ] \rightarrow (Had \cup Felt) \langle 1, 0.9 \rangle$$

Statement (26) expresses the disjunction in the premise. (27) indicates that he had the anesthetic and he did not feel pain. (28) provides that he felt pain and he did not have the anesthetic.

The problem here is that the conclusion ('he had the anesthetic or felt pain, or both') includes 'or both'. As we can derive from what was explained in the previous section, 'or both' is not acceptable. It is incompatible with what (27) and (28) capture.

If (27) is true, then (29) is true as well.

$$(29) \quad [ \{He\} \times (Pain - Anesthetic) ] \rightarrow (Had \cup Felt) \langle 0, 0.9 \rangle$$

On the other hand, if (28) is true, then (30) is also true.

$$(30) \quad [ \{He\} \times (Anesthetic - Pain) ] \rightarrow (Had \cup Felt) \langle 0, 0.9 \rangle$$

Given that (27) or (28) must be the case, and that (27), (28), (29), and (30) all have the same confidence value ( $c = 0.9$ ), the conclusion in (2) is hard to accept. The choice rule cannot help here either.

## CONCLUSIONS

This paper is not intended to compete with any of the frameworks in the literature dealing with inferences such as (1) and (2). The point here is just that those inferences cause logical difficulties. People tend to come to conclusions contrary to those in First-Order Predicate Calculus.

NAL is a logic being able to address them. In the case of (1), the concept of intensional intersection is crucial. Intensional intersection is isomorphic to disjunction. Therefore, the premise in (1) can be expressed as an intensional intersection in NAL. In the latter logic, the intensional intersection between two terms is a subset of each of the intensions of those terms.<sup>40</sup> This fact allows accepting the conclusion in (1).

A possible objection can be the possibility of the disjunction being interpreted as exclusive. In those cases, we need to resort to the concept of extensional difference. By means of that concept, we can express exclusive relations between disjuncts. Thus, we can show that, whenever disjunction is exclusive, there are paths with high confidence value preventing the conclusion in inferences with a structure akin to that of (1) from being accepted.

<sup>40</sup> *Loc. cit.*

The latter fact helps explain what happens with inferences with the structure of (2). If we use extensional difference to capture exclusivity, we can note that it is impossible to conclude the same disjunction interpreted as inclusive.

The arguments above are pieces of evidence in favour of the potential of NARS to come to the same conclusions as us.<sup>41</sup> Further studies in this direction can clarify that potential even more.

<sup>41</sup> Which does not mean that NARS necessarily thinks as a human being; see, e.g., P. Wang, *Non-Axiomatic Logic*.