MAIMON AND COHEN ON DIFFERENTIAL CALCULUS, TRANSCENDENTAL PHILOSOPHY AND METAPHYSICS[1]

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Abstract. Salomon Maimon and Hermann Cohen propose a profound modification of Kantian philosophy through a transcendental interpretation of differential calculus. This paper presents both theories and discusses their implications for the possibility of metaphysics.

Keywords: Cohen, Differential, Kant, Maimon, Metaphysics.

INTRODUCTION

In his famous letter to Marcus Herz from the year 1772, Kant claims that "the key to the whole secret of metaphysics" is contained in the following question: "What is the ground of the relation of that in us which we call "representation" to the object?"[2] Kant explains that the possibility of correspondence between the representation and the object can be clearly understood in two cases, depending on whether the representation is passive or active with respect to the object. In the case of merely passive representations, these involve only the way in which the subject is affected by the object. Therefore, the representation corresponds to the object as the effect corresponds to its cause. In the second case, if the representation created the object (as a divine intellect would do), the object would be the effect of the representation, and its conformity would also be understandable. However, Kant observes, the problem arises because the pure concepts of our understanding "are neither caused by the object nor do they bring the object itself

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[2] *Kants gesammelte Schriften* (AA), Königlichen Preußischen (Deutschen) Akademie der Wissenschaften. Berlin, 1902 ff., AA 10:130. Translations are taken from Imm. Kant, *Correspondence*, Cambridge, Cambridge University Press, 1999.

into being."[3] The peculiarity of our understanding is that it does not merely obtain matter for its representations from the sensible data, but spontaneously produces concepts all by itself. Since our understanding, through these representations, is not the cause of the object,[4] it is necessary to explain the objective validity of these a priori intellectual representations. In the *Critique of Pure Reason*, Kant characterizes this problem in legal terminology as the question *quid juris* and calls the explanation of how concepts can relate to objects a priori their transcendental deduction.[5] The result of this investigation is that a priori knowledge of objects is only possible as a priori knowledge of objects of possible experience.[6] Therefore, metaphysics in its first part, i.e., the *metaphysica generalis* or ontology, can take the safe course of a science, "where it concerns itself with concepts *a priori* to which their corresponding objects appropriate to them can be given in experience".[7] For this reason, one the one hand, "the proud name of an ontology, which presumes to offer synthetic a priori cognitions of things in general in a systematic doctrine (e.g., the principle of causality), must give way to the modest one of the mere analytic of the pure understanding."[8] On the other hand, the second part of metaphysics, i.e., the *metaphysica specialis* composed of theology, cosmology, and psychology, which aims at the supersensible, cannot reach its goal by means of speculative reason alone. Rather, the practical use of reason must also be taken into consideration, so that a priori knowledge beyond the limits of possible experience is possible, but only from a practical point of view.[9]

The Kantian answer to the *quid juris* question is developed in parallel with an account of the peculiarities of mathematical knowledge. In 1772, Kant concedes that in mathematics the correspondence of a priori concepts to objects is possible because, in this case, the objects are quantities and, as such, can be represented "by taking numerical units a given number of times."[10] But in the case of qualities, the correspondence of a priori concepts and objects "is still left in a state of obscurity".[11] In the mature doctrine of the first critique, Kant points out that only the concept of magnitudes can be constructed, i.e., exhibited *a priori* in intuition, while qualities can only be exhibited with the aid of experience. While quanta can be cognized a priori, "in all quality (the real of appearances) we can cognize a priori nothing more than their intensive *quantity*, namely that they have a degree,

[3] AA 10:130.

[4] Except for moral ends.

[5] A84f = B116f

[6] B165-166.

[7] BXVIII-XIX.

[8] A247 = B303.

[9] On this issue, see Hernán Pringe, "Crítica de la metafísica y metafísica crítica", in *Pensamiento*, vol. 78, 299, 2022, pp. 995–1004.

[10] AA 10:131.

[11] AA 10:131.

and everything else is left to experience."[12] As Kant notes, the shape of a geometric body (e.g., a cone) may be intuited a priori, according to its concept, but the color of that body must first be given empirically.[13] An intuition corresponding to the concept of reality [*Realität*] can only be obtained from experience, although we cognize a priori that the object of that empirical intuition will have a degree.

In this paper, I will discuss two approaches to the *quid juris* question and the problem of metaphysics that depart from the Kantian solution and rely heavily on a philosophical interpretation of mathematical knowledge: Salomon Maimon's and Hermann Cohen's idealism. Both Maimon and Cohen propose a philosophical theory of differentials that modifies the Kantian doctrine of the correspondence between concepts and sensible objects by reinterpreting the notions of extensive and intensive magnitudes. In this way, they challenge the Kantian answer to the problem of metaphysics.

1. MAIMON

In his *Essay on Transcendental Philosophy,* Salomon Maimon gives "a clue"[14] to the justification of the applicability of pure concepts to empirical objects, or, in other words, he puts forward his own answer to the question *quid juris*.[15] According to Maimon, Kantian philosophy cannot solve this problem, but a satisfactory solution may nevertheless be reached from a Leibnizian–Wolffian position.[16] More precisely, the heterogeneity that Kant establishes between understanding and sensibility should be rejected in favor of a homogeneity based on the common origin of concept and sensible data. Maimon claims that sensible data relate to a concept in the same way that the generated relates to its law of generation. At this point, Maimon introduces his theory of differentials to explain the empirical applicability of a priori concepts. The relation between a function and its derivative would be the same as that between the sensible and the intellectual law of its generation. Since this law can be determined, the sensible loses its character of being passively received and thus heterogeneous with the concepts of understanding. In this way, a satisfactory solution to the *quid juris* question can be reached.

[12] A176 = B218.

[13] A714 = B743.

[14] Salomon Maimon *Gesammelte Werke* [GW] (V. Verra, ed.). Hildesheim: Olms, 2003; GW II: 9. All translations are taken from Salomon Maimon, *Essay on Transcendental Philosophy*, translated by N. Midgley, H. Somers-Hall, A. Welchman, M. Reglitz, London/New York: Continuum, 2010.

[15] See Hernán Pringe, "La teoría de los diferenciales de Salomon Maimon, la pregunta quid juris y la posibilidad de la metafísica como ciencia", in *Anales del Seminario de Historia de la Filosofía* 33 (1), 2016, pp. 81–102.

[16] GW II: 64.

The *quid juris* question arises because categories and empirical intuitions are heterogeneous: how is it conceivable, Maimon asks, that *a priori* concepts of understanding can provide determinations of something *a posteriori*?[17] In view of this heterogeneity of intellectual rule and sensible data, Kant's strategy is to explain the applicability of categories to appearances by considering a mediating representation: the schema. Schemata are transcendental determinations of pure time that are homogeneous with the categories and with the appearances.[18] Since time is the form of all appearances in general, the determinations of pure time represented by schemata will be valid with respect to the matter given in that form. Moreover, since such determinations are determinations in accordance with the categories, the categories will be valid with respect to empirical intuitions. But, for Maimon, in this way Kant merely replaces one problem with another. If the initial difficulty was to explain how a priori concepts can be applied to empirical intuitions, we are now faced with the problem of explaining the applicability of a priori concepts to a priori intuitions. In fact, an a priori intuition, insofar as it is a sensible representation, is also heterogeneous with respect to the concepts of understanding. Therefore, the Kantian schematism of the categories is unfruitful.[19]

But Maimon's criticism does not stop there. Even if we accept for the sake of argument that the application of categories to the a priori form of all appearances (i.e., time) is unproblematic, we could only explain the application of categories to an object in general, not to any particular object. In the case of causality, Maimon points out that since the principle of causality does not contain the ground of what is particular in a particular causal law, what is particular in such a law has no ground for us at all.[20]

In order to guarantee the applicability of the concept of causality to a given object, it is not enough to maintain that such an object must have a cause, as the principle of causality claims. This condition must be satisfied in order for an object (or rather an event) to be represented as belonging to the objective series of experience and not merely to the subjective series of perception. Nevertheless, the cause must contain not only the ground of the formal aspect of the determinate object (i.e., its place in the ordered series of time), but also the ground of the material aspect of the object (i.e., its being so and not otherwise).[21] Kantian schematism explains the determination of pure time according to the categories, and in particular it establishes that the order of succession results from the

[17] GW II: 41; GW II: 363.
[18] B 177 – 178.
[19] GW II: 64.
[20] GW II: 393 – 394.
[21] In this sense, Maimon underlines that "a cause is something by means of which something else is determined. However, determined means not merely *posited*, but *determinately posited*". GW II: 94.

application of the category of causality to the pure manifold of time.[22] Thus, the cause of an event necessarily occurs prior to the event. Maimon's point is that this necessary condition for something to be a cause (i.e., for the category of causality to be applied to a determinate object) is not a sufficient condition. Not only must the effect follow the cause, but it must follow the cause in a determinate way. However, the mere structuring of pure time according to the categories leaves unexplained how a certain element can put a later element in a determinate way, for it does not explain how the cause can cause the being of the effect so and not otherwise.[23] In summary, we can accept that the Kantian doctrine accounts for the subsumption of an object in general under categories. Such an object will have only formal determinations, such as a place in the ordered temporal sequence. However, the Kantian doctrine does not account for the application of categories to determinate objects, because the structuring of pure time according to categories does not explain the possibility of the connection of appearances with respect to their matter. In order to justify the applicability of categories to determinate objects, the applicability of categories to the matter of appearances must be explained, since the Kantian account of their application to the form of empirical objects does not make their application to the matter of appearances comprehensible. Maimon claims that such an explanation is impossible if the matter of appearances has its origin in a receptivity independent of understanding. As we have seen, in this situation the categories can only refer to what is universal in the appearances and not to what is particular in them. Maimon claims that the applicability of the categories to determinate objects would only be guaranteed if the matter of appearances were also the product of a spontaneity. In that case, the categories would not connect a manifold alien to the understanding, but they would participate in the generation of such matter through spontaneity. If we adopt this point of view, the character of the given, which the matter of appearances has, is reinterpreted. The given is not what is passively received as the effect of an affection, but rather an element of cognition whose law of generation we ignore. [24] As we have seen, for Maimon the question *quid juris* does not have a satisfactory answer in the way Kant poses it. On the contrary, the problem of the applicability of categories would have a solution if we were to assume that the matter of appearances is the product of a spontaneity, i.e. if we were to assume that it is generated in accordance with a law and thus is not an absolute and irrational fact.[25]

[22] B184 – B185. On this issue, see Mario Caimi, "The Logical Structure of Time according to the Chapter on Schematism", in *Kant-Studien* 103, 2012, pp. 415–428.

[23] Here it is not demanded that an empirical connection be established a priori, but rather that the *possibility* of that connection be determined. This must be done not only regarding the form of the connection but also regarding its matter.

[24] GW II: 415 – 416.

[25] GW II: 64.

The differential is precisely the generating rule of the sensible. If, in an empirical representation, all synthesis according to categories were suspended, the Kantian doctrine establishes that we would obtain an ultimate non-intellectual element (i.e., the sensible manifold), which would constitute the matter of synthesis, the conceptual form of which we have completely eliminated. On the contrary, for Maimon, the limit of a complete suspension of the action of our understanding would not be a non-conceptual element, but rather the intellectual rule (albeit unknown to us) according to which the empirical representation is generated. The relationship that philosophy establishes between the empirical content and its law of generation is, for Maimon, analogous to the relationship that exists in mathematics between a function and its differential. The empirical content is the product of a synthesis, just as a given differentiable function results from the integration of its differential. In view of this analogy, what is merely given to our understanding must rather be conceived as produced according to a rule.

Both Kant and Maimon maintain that the task of understanding is to unify a manifold by subsuming it under a concept. But according to Maimon, and in contrast to Kant, this can only be done if the manifold is the product of a spontaneity, so that its unification is achieved by pointing out the way in which the manifold is produced.[26] While for Kant the concept unifies a passively received manifold, for Maimon the concept unifies a spontaneously produced manifold, even if our understanding is not fully conscious of the rule of this production. Such a rule is a condition of the intellectual syntheses that determine appearances as extensive and intensive magnitudes.[27] Maimon maintains that the intellectual action that produces the unity of intuition is exercised on an element that must be conceived as originating from an unlimited spontaneity, although the generating rule of the manifold remains unknown to our finite understanding. This rule is the differential. The differential is the limit of a complete suspension of all aggregation and coalition syntheses.[28] This limit cannot be reached by us, because it implies an infinitely decreasing series until the complete disappearance of consciousness.[29] Thus, the differential is an idea that cannot be exhibited in intuition and it is rather a *noumenon*, i.e., an object beyond sensibility.[30]

The theory of differentials allows Maimon to explain the applicability of categories to determinate empirical objects. In contrast to Kantian schematism, categories are not applied to the form of sensibility (time and space) and therefore to the matter of intuition. Rather, categories would be applied to certain intellectual elements (the differentials) so that pure concepts could thereby subsume empirical objects under them. In this case, the *quid juris* question would not arise, because

[26] GW II: 32 – 33.
[27] GW II: 27 – 30.
[28] B201n.
[29] GW II: 349.
[30] GW II: 32.

the categories would not be applied to representations heterogeneous with the understanding: spontaneity would exercise its synthesis according to the categories on its own products (the differentials). With his theory of differentials, Maimon reformulates the Kantian doctrine of knowledge. Sensibility provides the matter of cognition, but not because this matter is passively received. What we call sensible data is rather the product of a spontaneity, although the rule of its generation remains unknown to us. The sensible given as such is not an object of consciousness. Consciousness arises as a result of a synthesis carried out by the imagination, which produces the unity of the manifold of intuition. The understanding applies the categories to the differentials and thus establishes the connections between the appearances whose rules of generation the differentials express. From the point of view of our finite intellect, therefore, the "subjective" order of the "operations of the mind" is:[31] sensibility (sensation), intuition, concepts of understanding, and finally ideas of reason, by means of which the totality of the cognitions of understanding is sought. But if we adopted the perspective of an unlimited intellect, in the order mentioned above sensibility and intuition would disappear, because such an intellect would know the rule of generation of what we call the given. From this point of view, the order of the operations of the mind would be: ideas of understanding (differentials), concepts of understanding, and ideas of reason. These three operations are "conditions of intuition itself".[32] Maimon calls this order the "objective" order.

While for Kant the categories are applied to empirical objects by means of schemata, Maimon maintains that empirical objects are subsumed under the categories when the understanding thinks the differentials of the objects according to the categories:

> "If one thus judges that fire melts wax, then this judgement does not relate to fire and wax as objects of intuition, but to their elements, which are thought by the understanding in the relation of cause and effect to one another."[33]

If we think the differentials of the heat of the fire and the degree of fluidity of the wax according to the category of causality, then the understanding establishes a functional dependence between the temperature of the fire and the state of aggregation of the wax.[34] In this way, the rule of the emergence of the heat of the fire (i.e., the way in which its temperature changes) is correlated with the rule of

[31] GW II: 81. GW II: 376 – 377.

[32] GW II: 82. Therefore, the categories are not merely conditions of the possibility of experience (as Kant maintains), as though there could nevertheless be perception without them. Rather, for Maimon categories are "conditions of perception itself" (GW II: 215), because intuitions could not be given independently of the categories. In contradistinction to this, Kant claims that "intuition by no means requires the functions of thinking." (A91/B123).

[33] GW II: 356.

[34] The categories are "rules of the way in which objects arise" (GW II: 277).

the variation of the aggregation state of the wax. The understanding connects the representation of fire and wax as cause and effect, but not merely in terms of their temporal form, as Kantian schematism would suggest. In such a case, the intellectual synthesis would only establish the objective temporal order of experience as heat of the fire-melting of the wax, but it would remain undetermined why it is the heat of the fire and not some other simultaneous event that causes the wax to melt. Now it is rather the matter of representations, what is connected by means of the relations of the differentials. In this way, the heat of the fire and the melting of the wax are synthesized as cause and effect, because the generation of the content of these appearances, and not only their place in time, is set as the condition and the conditioned. According to Maimon, the category of causality is applied directly to the particular matter of an empirical object and not to the universal form that makes the object an object of experience in general. Since there is no heterogeneity between the matter of appearances and the categories, for both are products of spontaneity, the *quid juris* question does not arise.

Maimon states that he agrees with Kant on the definition of metaphysics. Metaphysics, Maimon claims, is the science of things in themselves.[35] For Kant, things in themselves are the substrata of their appearances, and the former are completely heterogeneous with respect to the latter.[36] Thus these supersensible substrata remain inexorably unknown to us, for appearances give us knowledge only of the way in which we are affected, but not of things as they might be beyond their being given by sensibility. Therefore, metaphysics as knowledge of the supersensible is impossible for Kant. For Maimon, on the other hand, the science of things in themselves is not the science of something beyond appearances, but the science of the rules of the arising of appearances. These rules are not objects of intuition and, in this sense, are things in themselves. However, these rules can be thought of in a determinate way precisely by means of the intuitions that arise from them. Mathematics, and in particular differential calculus, shows us that this is possible. This is the fact upon which the possibility of metaphysics as a science can be understood.

According to Maimon, there is no heterogeneity between the sensible and its intelligible ground precisely because the latter is the rule of the arising of the former. When we intuit an object, we represent it as having already arisen. When we think of it, we do not represent it as having already arisen, but as arising.[37] The rule of the arising of an object is its differential.[38] Although the differential is not an object of intuition, it can be known by its sensible image or schema.[39]

[35] GW III: 200.
[36] GW III: 200.
[37] GW II: 33.
[38] GW II: 33 and 28 footnote.
[39] GW II: 366.

Consider, for example, a body moving in space. Its trajectory is the sensible image of its instantaneous velocity, for the rule of the arising of the trajectory is precisely the instantaneous velocity of the body. However, the instantaneous velocity cannot be known directly, but only by its effect, i.e. by the space which the body would traverse in a given period of time if it moved constantly at that velocity.[40] As rules for the arising of trajectories, instantaneous velocities "are mere limit concepts [*Gränzbegriffe*], which we can approach nearer and nearer to, but never reach. They arise through a continuous regress or through the diminution to infinity of the consciousness of an intuition."[41] This reduction of intuition to an infinitely small space and an infinitely small time is what we would call, in modern terms, the limit of average velocity when the time interval tends to zero.[42] Thus, although the instantaneous velocity itself is not an object of intuition, it can be cognized by means of its schema or sensible image: the trajectory. More precisely, the instantaneous velocity is the derivative of the trajectory. Thus, differential calculus teaches us how to determine the intelligible ground of a sensible object, and in this way it provides "the explanation of the possibility of a metaphysics in general, through the reduction of intuitions to their elements, elements that I call ideas of the understanding [*Verstandsideen*]".[43] These ideas of understanding, i.e., the differentials, are the *noumena* behind the *phenomena*. Such an idea does not correspond to a transcendent reality, but rather to a *method* "for finding a passage from the representation or concept of a thing to the thing itself; it does not determines any object of intuition but still determines a real object whose schema is the object of intuition".[44] Since the differential is the rule of the arising of the sensible object and the latter is the key to the cognition of the former, it can be said that the differential is the *ratio essendi* of the sensible object, while the sensible object is the *ratio cognoscendi* of the differential.[45]

Whereas differential calculus shows us how to go from the sensible to the supersensible, integral calculus teaches us how to make the inverse transition, i.e., how to go from the supersensible to the sensible. In our example, the trajectory is the integral of the instantaneous velocity. Maimon transforms the mathematical

[40] GW II: 290 – 291.

[41] GW II: 28 footnote.

[42] Neither the duration of the movement nor the space traversed belong to the essence of the *instantaneous* velocity.

[43] GW II: 9.

[44] GW II: 365.

[45] Maimon identifies *Entstehungsart* and *Essentia realis* in GW II: 415. Although Maimon later characterizes differential calculus as a method of fictions (see, e.g., GW IV: 51f.; GW V: 263 – 264), he does not make this characterization in his *Essay on Transcendental Philosophy*. For a critical evaluation of the interpretations of Maimon as a fictionalist, see Achim Engstler, *Untersuchungen zum Idealismus Salomon Maimons*, Stuttgart-Bad Canstatt: Fromann Holzboog, 1990, 139f. In any case, fictional, as Freudenthal rightly points out, is not to be opposed to real, but to actual. See Gideon Freudenthal, "Maimon's Philosophical Program. Understanding versus Intuition", in *Internationales Jahrbuch des Deutschen Idealismus*, 8, 2010, pp. 83–105, p. 97.

problem of the relationship between integrals and differentials into the metaphysical problem of the relationship between the sensible and the supersensible. Differentiation and integration are the opposite directions in which the transition between the sensible and the supersensible can take place. While differentiation takes us from the sensible to the supersensible, from the phenomena to the noumena that ground them, integration takes us the other way around, from the supersensible to the sensible, from noumena to phenomena:

> "So the result of the theory is the following. With Kant, I maintain that the objects of metaphysics are not objects of intuition and cannot be given in experience. But I depart from him in this respect: he claims that they are not objects that can be thought by the understanding as determined in any way; by contrast, I hold them to be real objects, and although they are in themselves only ideas, they can nonetheless be thought as determined by means of the intuitions that arise from them. Further, just as we are in a position to determine new relations between magnitudes themselves by reducing them to their differentials (and these in turn to their integrals), so by reducing intuitions to their elements, we are in a position to determine new relations between them, and in this way to treat metaphysics as a science."[46]

Although the objects of metaphysical cognition are not sensible objects, the understanding can determine them by means of the intuitions that arise from them. Differential calculus shows that this is indeed possible, and therefore that cognition of the supersensible is possible, if only we conceive of the latter not as a transcendent sphere, but as the realm in which the rules of the genesis of the sensible are to be found.

2. COHEN

In his *Logic of Pure Knowledge*, Hermann Cohen seeks to establish the foundation of metaphysics as a system of philosophy composed of four members: logic, ethics, aesthetics, and psychology. This logic of pure knowledge "has always been metaphysics", because, as the first part of the system, it provides metaphysics with its foundations.[47] The key to this logic is Cohen's interpretation of the infinitesimal. Cohen claims that Kant did not properly understand the importance of calculus for transcendental philosophy, and therefore was unable to recognize the productive capacity of thought. For Cohen, "if the infinitesimal principle had found the place it deserved in the *Critique*, sensibility would not have been placed before thought [and] pure thought would not have been undermined in its

[46] GW II: 195–196.

[47] Hermann Cohen, *Logik der reinen Erkenntnis. System Der Philosophie*, 1, 1. Aufl., Berlin, 1902, p. 516 (our translations).

autonomy"[48]. Kant did not see the importance of the infinitesimal principle. In contrast, the logic of pure knowledge emphasizes "the crucial logical significance of the infinitesimal principle" and can even be characterized as "the logic of the principle of infinitesimal calculus"[49]. According to this principle, thought produces its own content by itself, i.e., by means of the infinitesimal calculus, thought produces the matter that for Kant could only be given by sensibility.

Cohen's approach to differential calculus evolved from a first formulation contained in *The Principle of the Infinitesimal Method and its History* of 1883 and the second edition of *Kant's Theory of Experience* of 1885, to the final expression of the *Logic of Pure Knowledge* of 1902. In the first exposition of his doctrine, Cohen explores the relationships between the concepts of infinitesimal, extensive magnitude, intensive magnitude, and reality [*Realität*]. In doing so, he attempts to demonstrate the productive character of intensive magnitude and, more generally, the productive character of thought.

As it is well known, Kant distinguishes two different kinds of combination (*conjuctio*). These are: a) composition (*compositio*) and b) connection (*nexus*). Composition is the synthesis of a manifold of what *does not necessarily* belong *to each other*. This is the synthesis of the *homogeneous* in everything that can be considered mathematically. Composition can in turn be further subdivided into the synthesis of *aggregation*, directed toward *extensive* magnitudes, and that of *coalition*, directed toward *intensive* magnitudes.[50] The concept of a magnitude (*Quanti*), Kant explains, is the consciousness of the homogeneous manifold in intuition in general, insofar as it is through it that the representation of an object first becomes possible.[51] Extensive magnitude is the magnitude in which the representation of the parts precedes the representation of the whole and makes it possible. The apprehension of extensive magnitudes is always successive, i.e. it takes place according to a successive synthesis of a manifold of antecedently given parts. To the contrary, intensive magnitude is that which can only be apprehended as a unity.[52] In this case, the apprehension is not successive but instantaneous. There is no manifold of antecedently given parts, but just a multiplicity that can only be represented through approximation to negation "= 0."[53] Every reality in the appearance has intensive magnitude, i.e., a degree.

Extensive magnitudes correspond to the form of intuition, i.e., the pure space-time manifold of appearances, while intensive magnitudes correspond to the

[48] *Ibidem*, p. 32.
[49] *Ibidem*, p. 31.
[50] A162/B201 footnote.
[51] A162/B203.
[52] On this definition, Cohen claims: "Fehlerhaft bleibt die unabgelöste Beziehung auf die subjectiven Mittel des Bewusstseins, hier also auf die Apprehension." (Hermann Cohen, *Kants Theorie der Erfahrung*, 2. Aufl., Berlin, 1885, p. 427).
[53] A168/B210.

matter of intuition, i.e., sensation. The spatiotemporal pure form of appearances is apprehended successively, while their empirical matter is apprehended instantaneously. This difference in the apprehension of the form and matter of appearances has profound consequences for the Cohenian interpretation of extensive and intensive magnitudes.

Extensive magnitudes are comparative magnitudes (*Vergleichungsgrössen*), because their quantity is obtained by comparison with a certain unit of measurement. Since this unit is arbitrary, conventional and depends on the peculiarity of our senses,[54] Cohen claims that the object of the extensive magnitude, as a mere result of a synthesis of aggregation, lacks a proper foundation.[55] Thus, Cohen argues that the comparison of an object with the unit of measurement always presupposes something present in itself and by itself, which constitutes the necessary foundation, and which does not depend on the unit of measurement adopted. The ground of the extensive magnitude is not the relative and arbitrary unit of comparison, but an absolute unity that will be found not in pure intuition, but in the *real* in space and time.

If a totality is successively apprehended to be represented as an extensive magnitude, then there must be something real to be apprehended at each moment. This instantaneous unity is the unity of intensive magnitudes, which is therefore an absolute unity, "for as contained in one moment no representation can ever be anything other than an absolute unity."[56]

For Cohen, this non-extensive ground of extension is the *intensio*.[57] More precisely, Cohen claims that the intensive, absolute, instantaneous unity of the real in space and time is the *infinitesimal*.[58] Accordingly, differential calculus is the method by means of which intensive magnitudes ground extensive ones.[59] For

[54] Hermann Cohen, *Das Princip der Infinitesmal-Methode und seine Geschichte. Ein Kapitel zur Grundlegung der Erkenntnisskritik*, Berlin, 1883, p. 70.

[55] Hermann Cohen, *Kants Theorie der Erfahrung*, p. 428.

[56] A 99. "Denn wenn die Einheit einer Vielheit gedacht werden soll, so muss vor Allem die Einheit selbst gedacht sein. Das ist die Einheit, die nur als Einheit "apprehendirt" wird." (Hermann Cohen, *Kants Theorie der Erfahrung*, p. 428). In this connection, Böhme claims: "Wir glauben nach dieser Analyse das Recht zu der Behauptung zu haben, daß das Grundmaß als Größe intensiv erfahren wird […]. Die extensive Größenschätzung setzt die intensive als Beurteilung des Grundmaßes voraus." (Gernot Böhme, „Über Kants Unterscheidung von extensiven und intensiven Größen", in *Kant-Studien* 65, 1974, pp. 239–258, p. 248).

[57] Hermann Cohen, *Das Princip der Infinitesmal-Methode und seine Geschichte. Ein Kapitel zur Grundlegung der Erkenntnisskritik*, p. 71.

[58] Hermann Cohen, *Kants Theorie der Erfahrung*, p. 427. This doctrine was not present in the first edition of *Kant's Theory of Experience*. In 1871, Cohen rather claims: "Das Reale der Empfindung ist diejenige Grösse, die "nur als Einheit apprehendiert wird": es ist die Reizeinheit, zu welcher wir die Empfindung objectivieren." (Hermann Cohen, *Kants Theorie der Erfahrung*, 1. Aufl., Berlin, 1871, p. 216). See Scott Edgar, "Leibniz's Influence on Hermann Cohen's Interpretation of Kant", in *Kant e-Prints*, série 2, v. 16, n. 2, 2021, pp. 200–230.

[59] In this sense, Kant states: "Qualitative Einheit ist wie der Grund des Ganzen, quantitative wie ein Theil des Ganzen zu betrachten." (AA 18: 322). Quoted in Hermann Cohen, *Kants Theorie der Erfahrung*, 1875, p. 431.

example, the trajectory of a body x(t) moving at a given instantaneous velocity v(t) is obtained by integrating the velocity v(t) from the initial $t = t_0$ to the final $t = t_f$. Thus, velocity, as an intensive magnitude, grounds trajectory, as an extensive magnitude.[60]

Cohen maintains the identity between infinitesimal and intensive magnitudes and claims that this identity was universally accepted in Kant's time[61]: the critical doctrine does not bring any new thesis in this respect. For Cohen, the Kantian novel viewpoint is rather the identification of the infinitesimal with the real [*das Reale*] in space and time. However, Cohen argues that reality is not grounded in sensation, as Kant claims, but only in thought. To be more precise, the principle of the infinitesimal method is a principle of thought that provides the pure spatiotemporal extensive magnitude of appearances with real intensive magnitude by generating the former from the latter, just as the trajectory of a body is generated from its instantaneous velocity.

This generation produces space and time as *quanta continua*, that is, as magnitudes of which no part is the smallest or simple. Cohen points out that Kant himself claims this generation of the extensive magnitude from the intensive in Newtonian terms in the following passage[62]:

> "Space and time are quanta continua. […] Magnitudes of this sort can also be called flowing, since the synthesis (of the productive imagination) in their generation is a progress in time, the continuity of which is customarily designated by the expression "flowing" ("elapsing")."[63]

According to Newton's terminology, the fluent is the extensive magnitude, generated by the fluxion as an intensive magnitude. Thus, space and time are produced as quanta continua by a condition of thought: reality as an intensive magnitude.[64]

[60] In this regard, see AA 14: 496 and AA 4: 493–494.

[61] See Hermann Cohen, *Das Princip der Infinitesmal-Methode und seine Geschichte. Ein Kapitel zur Grundlegung der Erkenntnisskritik*, p. 14; Hermann Cohen, *Kants Theorie der Erfahrung*, 1875, p. 427.

[62] In Newton's terms, the fluent is the extensive magnitude and the fluxion is the intensive one.

[63] A170/B211-212. See also AA 18: 167 and AA 14: 53. On Kant's reception of Leibnizian differentials and Newtonian fluxions, Büchel claims: "Mit dem Newtonschen Konzept der Fluxion als Geschwindigkeit des Fallens oder Steigens ist wegen der Geschwindigkeitsvorstellung eine Zeitvorstellung nahegelegt, wie es für das Erzeugen einer Linie durch Fluxion dargelegt worden ist. Dadurch ist das Newtonsche Konzept der Grundlegung der Differentialrechnung für die mathematische Beschreibung „fließender" Bewegungsabläufe in der Zeit geeigneter als Leibniz' Konzept der „unvergleichbarkleinen Größe", der Differentialgröße, die von Leibniz selber schon in die Nähe der unendlich kleinen Größen gerückt wurde." (Gregor Büchel, *Geometrie und Philosophie*, Berlin: De Gruyter, 1987, p. 242).

[64] In the same sense, Maimon states that the extensive magnitude is the schema of the intensive magnitude, and that the intensive magnitude is the differential of the extensive, while the extensive is the integral of the intensive. GW II: 121–122. Schulthess underlines that also Lotze and Trendelenburg claim that extensive magnitudes are generated from intensive ones (Peter Schulthess, *Einleitung*, in Hermann Cohen, *Das Prinzip der Infinitesimal-Methode und seine Geschichte*, Hildesheim/Zürich/ New York: Olms, 1984, pp. 7–46, pp. 24–25).

As we have said, Cohen criticizes Kant for maintaining a false relation between reality as intensive magnitude and sensation. The infinitesimal as intensive magnitude grounds the objective character of spatiotemporal magnitudes by generating them according to the rules of differential calculus. The intensive magnitude of reality, as infinitesimal, is the unity of generation of the finite.[65] But this infinitesimal unity, Cohen claims, has its basis only in thought, not in intuition or sensation. Space and time are the forms in which the real is ordered, but this real is not passively received but spontaneously produced by thought.

Thus, according to Cohen, differential calculus reveals the productive character of intensive magnitude and, more generally, the productive character of thought. For Cohen, this means that the Kantian *critique* of knowledge should be transformed into a *logic* of *pure* knowledge. In this transformation of transcendental idealism, Cohen will follow the lines established by Kant, only to go beyond the point Kant reached.

For Cohen, thought is pure because there is no manifold that is given to it. While the Kantian synthesis is to be performed on the spatiotemporal manifold provided by sensibility, pure thought does not depend on sensible data. Pure thought does not merely unify a given manifold but produces both the unity of the manifold and the manifold to be unified. To capture this productive capacity of thought, Cohen uses the term *Erzeugen*. Pure thought is generation.

Even if the Kantian concept of synthesis does not reach the complete autonomy of pure thought, the productive character of thought, which does not presuppose an intuitive manifold, is still discernible in the Kantian discussion of intensive magnitudes. The real that fills time is represented as a quantum by thinking its continuous and uniform generation in time.[66] We have already seen that Cohen understands this intensive magnitude as an infinitesimal magnitude, but criticizes the relation that Kant establishes between sensation and reality. There are two other points to emphasize now. The first is that thought produces not only unity, but also the manifold to be unified. The real is immediately given as an absolute unity. As such, it contains no diversity. Only when the real is thought as the product of a continuous and uniform generation in time can a manifold be attributed to it, namely the manifold of intermediate states between the complete absence of the real and its presence.[67] The real, as a quantum, will be conceived as the unity of the multiplicity of gradations through which it has passed in its

[65] Hermann Cohen, *Das Princip der Infinitesmal-Methode und seine Geschichte. Ein Kapitel zur Grundlegung der Erkenntnisskritik*, p. 146.

[66] B183.

[67] See Böhme, *op. cit*., p. 250. Caimi claims that this synthesis takes place instantaneously: "Nicht im Laufe der Zeit (nicht in einer Mehrzahl von unterschiedlichen, nacheinander folgenden Augenblicken), sondern in jedem nennbaren Augenblick, „in einem Punkte und in einem Augenblicke" (*Prol*, AA 4: 309, Anm.) der Zeit erfolgt die Synthesis der Realität." (Mario Caimi, "Das Schema der Qualität bzw. der Realität", in Dieter Hüning, Carsten Olk, Stefan Klingner (eds.), *Das Leben der Vernunft*, Berlin, De Gruyter, 2013, pp. 95–108, p. 98).

generation. This gradation is not given: it is a product of thought. The manifold to be unified is thus generated by spontaneity.

The second point is that the generation of the real, whose multiplicity can only be represented by an approximation to negation, is a production from a negation. Cohen emphasizes that this negation is not a *nihil negativum*, but a *nihil privativum*, understood as a limit. For this reason, the disappearance of sensation does not correspond to pure intuition, as Kant states,[68] but to the origin of sensation.[69] The concept of origin, already present in Cohen's interpretation of Kant, will become the central concept of the logic of pure knowledge.[70]

In the infinitesimal synthesis corresponding to the Kantian categories of quality the productive character of thought is revealed. Thought does not merely unify a given manifold but produces it: thought is generation [*Erzeugen*][71]. Moreover, this generation takes place *ab nihilo*:[72] thought is origin [*Ursprung*]. In his *Logic*, Cohen understands these aspects of the Kantian synthesis that grounds the quality of the *object* as moments of the quality of *thought* itself.

The fundamental way in which thinking proceeds is called the quality of pure thought.[73] The most basic structure of pure thought is expressed not by categories, but by judgments corresponding to the laws of thought [*Denkgesetze*]. These are identity, contradiction, and origin. The law of thought of origin is the logically first one.[74] The judgment of origin expresses that law of thought by which a determination becomes a valid one for thought.[75] The judgment of identity expresses that law of thought by which such a determination is fixed, i.e., it affirms a content A obtained by continuous generation. Finally, the judgment of contradiction prevents the negation of such a content A from becoming a valid content of thought.[76] In the words of Parmenides, the judgments of identity and contradiction state that only Being exists and that Not-Being is not.[77]

For Cohen, the judgment of origin is an infinite judgment. Kant distinguishes between affirmative, negative and infinite judgments according to their quality. An affirmative judgment attributes a predicate to a subject, while a negative judgment asserts that a predicate does not belong to it. Conversely, an infinite judgment affirms a negative predicate. For example, in the judgment “the soul is immortal”, the non-mortality of the soul is positively predicated. For Cohen, the most basic

[68] B208.
[69] Hermann Cohen, *Kants Theorie der Erfahrung*, p. 435.
[70] Hermann Cohen, *Logik der reinen Erkenntnis. System Der Philosophie*, p. 32.
[71] *Ibidem*, p. 49.
[72] *Ibidem*, p. 70.
[73] *Ibidem*, p. 101.
[74] It is “das Denkgesetz der Denkgesetze” (*ibidem*, p. 100).
[75] *Ibidem*, p. 99.
[76] In this sense, the judgment of contradiction is “ein Urtheil vor dem Urtheil” (*ibidem*, p. 88).
[77] *Ibidem*, p. 94.

operations of thought are those of separating and unifying. A judgment expresses the correlation of these operations. In the case of the judgment of origin as an infinite judgment, this correlation is paradigmatically exemplified by the infinitesimal method. Cohen points out that the infinitesimal is not only inextensive, but also intensive. As inextensive, the infinitesimal is the negation of the extensive, while as intensive, the infinitesimal has a tendency toward extension.[78] As the origin of extension, the infinitesimal is the non-extended that produces extension, or in other words, it is the non-finite that produces the finite. The underlying structure of this relationship between the finite and the infinitesimal is that of infinite judgment. In the act whose law is expressed in the judgment of origin, thought separates the extended from the non-extended and at the same time unifies the two by making the non-extended intensive, i.e., the ground of the extended.

The logically first operation of pure thought is the separation and unification of a content and its ground of determination. The content is not yet conceived as a determinate A, but only as a determinable X. The X is only a determinable something.[79] The ground of determination is separated from this X as its negation. This relative negation is to be distinguished from the absolute negation contained in the judgment of contradiction. The first negation aims at the generation of an X and corresponds to a "Ursprungs-Etwas"[80]. This "Etwas" is designated by the noun *nothing* [*Nichts*]. Contrary to this, contradiction is an absolute negation which prevents any content from arising. It is the mere *no* [*Nicht*] as activity.[81]

The judgment of origin separates and at the same time unifies something with its relative nothing. This unification is not a mere static correlation between Being and Not-Being, but the continuous generation of the former from the latter. Relative nothingness is the transition point through which thought passes in order to generate the determinable.[82] Therefore, relative nothingness provides thought with a "springboard" to carry out generation.[83] In the relative nothing it is possible to conceive the foundation of the determinable something, just as in the complete elimination of extension the intensive was conceived as the foundation of the extensive.

Pure thought must generate its own content to turn into pure knowledge, i.e., into the thought of *being*. The judgments of the law of thought establish the logical structure of the determination of being. This determination may be conceived of as a series. By means of the judgment of origin, the general term X of this series is

[78] Hermann Cohen, *Das Princip der Infinitesmal-Methode und seine Geschichte. Ein Kapitel zur Grundlegung der Erkenntnisskritik*, p. 137.
[79] Hermann Cohen, *Logik der reinen Erkenntnis. System Der Philosophie*, p. 68.
[80] *Ibidem*, p. 87.
[81] *Ibidem*, p. 88.
[82] *Ibidem*, p. 76.
[83] *Ibidem*, p. 77.

posited, but yet no individual term A. The first step of this progressive process of pure knowledge is rather taken by the judgments of mathematics.[84] The judgments of mathematics posit the content of knowledge (that for Kant must be given by sensibility) as a product of pure thought.[85] The judgments of mathematics show how pure thought generates being by means of infinitesimal analysis. The first judgment of mathematics, the judgment of reality, posits the infinitesimal as the real in cognition.

According to the Kantian doctrine, reality is "that in empirical intuition which corresponds to the sensation."[86] In this connection, Kant even claims that "*Sensatio* [est] realitas phaenomenon."[87] In contrast, Cohen maintains that reality does not depend on sensation. Infinitesimal reality and not sensation is the ultimate condition of the *content* of thought and, in this way, of knowledge. For Cohen, the differential (dx) represents *what it is* [*das Seiende*], and only through it may that something (x) of which the differential is differential, in general, *be.*[88] Only through the infinitesimal may thought be the thought of being. But such infinitesimal has its origin in thought itself and not in any alleged receptivity.

While the judgment of origin claims that "thought must begin with the origin, as far as thought is thought of knowledge and therefore of being," [89] the judgment of reality establishes that such being is an infinitesimal reality. The infinitesimal is the non-sensible ground of the finite.

According to Kant, the spontaneity of thought unifies the manifold given by sensibility. In this act of synthesis, the understanding imposes its laws on appearances since this synthesis takes place in accordance with the categories. In contrast, for Cohen the object of knowledge conforms to pure thought because pure thought produces it. The Kantian category of reality determines a content independently given by sensibility. The category of reality is applied to the *received* matter that "fills time."[90] In contrast, through Cohen's judgment of reality, thought *produces* being as an infinitesimal reality.

[84] The four kinds of judgments of pure thought are the judgments of the laws of thought (quality), the judgments of mathematics (quantity), the judgments of mathematical natural science (relation) and the judgments of methodology (modality).

[85] See Hernán Pringe, "Cohen's Logik der reinen Erkenntnis and Cassirer's Substanzbegriff und Funktionsbegriff", in *Kant Yearbook*, 2020, pp. 137–168.

[86] A168/B209. See also A143/B182. Translation taken from Imm. Kant, *Critique of Pure Reason* [Kritik der reinen Vernunft], Cambridge, Cambridge University Press, 1998.

[87] A146/B186.

[88] Hermann Cohen, *Logik der reinen Erkenntnis. System Der Philosophie*, p. 114. Against Cohen, Natorp points out: "Aber es [das x] muss allerdings, und wäre es nur als Fragezeichnen, im Denken augestellt sein, ehe von dx überhaupt mit sicherem Sinn die Rede sein kann; und die Beziehung auf das x als das Zuerzeugende ist von dessen Begriff allerdings unabtrennbar." (Paul Natorp, „Zu Cohens Logik", in Helmut Holzhey, *Cohen und Natorp*, 2 vol., Basel/Stuttgart, 1986, pp. 43–78, p. 53).

[89] Hermann Cohen, *Logik der reinen Erkenntnis. System Der Philosophie*, p. 113.

[90] A143/B183.

CONCLUSIONS

Maimon and Cohen reject the Kantian distinction between sensibility and understanding. For Maimon, sensibility is incomplete understanding.[91] For Cohen, pure thought is the single condition of knowledge. Since knowledge is the thought of being, all determinations of being must be fully grounded in pure thought. For both, differential calculus shows the productive character of thought. [92] As Maimon puts it, in differential calculus, "the faculty of thought produces both the *form* and the *matter* of its thinking out of itself"[93]. In this respect, we are like God.[94] In Cohenian terminology, the infinitesimal method describes how the ideal becomes real through the pure generation of being.

In some places, Maimon calls "differential" not only the *rule* of the generation of the sensible, but also the *element* or smallest unit of the sensible.[95] The inconsistency that this double characterization seems to imply can be avoided by noting that, as we have shown, Maimon distinguishes two different perspectives from which the operations of the mind should be considered.[96] What is regarded as an element of the sensible from the subjective point of view is its rule of generation from the objective perspective.[97] However, one can still object that the rule of generation of the sensible is not described by the differential *dx*, but by the so-called differential quotient *dx/dy*, because the differential has no mathematical meaning by itself. Indeed, Russell argues against Cohen that the differential quotient *dx/dy* "is not a fraction, and *dx* and *dy* are nothing but typographical parts of one symbol."[98] In this respect, Cohen claims that the origin of the finite is only *defined* in the mere

[91] GW II: 183.

[92] In *The Principle of the Infinitesimal Method and Its History* of 1883, Cohen does not mention Maimon's theory of differentials. However, two years later, in the second edition of *Kant's Theory of Experience*, Cohen notes that Maimon has emphatically emphasized the connection between consciousness and differential in his investigations on transcendental philosophy (p. 423, footnote). Kuntze finds it remarkable that Cohen does not mention Maimon in his 1883 book, although Cohen reformulates the Kantian principle of the anticipations of perception "in exactly the same way, with exactly the same means, and with exactly the same systematic purposes as Maimon" (Friedrich Kuntze, *Die Philosophie Salomon Maimons*, Heidelberg: C. Winter, 1912, p. 339). See also Friedrich Kuntze, „Salomon Maimons theoretische Philosophie und ihr Ort in einem System des Kritizismus", in *Logos*, 3, 1912, pp. 285–308, p. 301. Cohen retorts that Kuntze has overlooked the reference made in the second edition of *Kant's Theory of Experience* and claims that Kuntze exaggerates his relationship to Maimon (p. 540, footnote).

[93] GW II: 2.

[94] GW IV: 42. See also GW V: 324, where Maimon claims that differential calculus is a "sparkle of divinity" and a "patent of nobility" that proves the lineage of human spirit from "pure intelligences".

[95] See, e.g., GW II: 29 footnote.

[96] GW II: 81 – 82 and 376 – 377.

[97] See A. Engstler, *op. cit*., p. 169.

[98] Bertrand Russell, *The Principles of Mathematics*, Cambridge, Cambridge University Press, 1903, p. 342. Russell makes no reference to Maimon.

differential, whereas by means of the differential quotient it can rather be *determined*,[99] but he does not explain this contrast any further. It is Dimitry Gawronsky, Cohen´s student, who provides an answer to one of the most common objections to Cohen's doctrine: that it wrongly gives priority to the differential over the differential quotient.[100] For Cohen, reality means that which is thought to exist in and for itself, without being articulated or ordered with others.[101] Therefore, any concept of relation presupposes the position of reality carried out by pure thought.[102] In particular, the concept of differential quotient presupposes the concept of the differential. Cohen emphasizes that the differential *dx* does not relate to another differential *dy* in the first place, but rather to the magnitude *x* generated from it. For Gawronsky, the differential is the logical ground of the finite, while the differential quotient enables the arising of the finite by providing the determinate law of this generation. Thus, the priority of the differential is not to be understood in a mathematical sense, as if one were claiming that *dy/dx* was a proper quotient. Rather, Gawronsky argues from a transcendental viewpoint that the differential quotient is nothing but the law of the generation of the finite from the differential. The differential is the origin of the finite, which arises according to the law given by the differential quotient.[103]

Both Maimon and Cohen derive consequences for the possibility of metaphysics from their revision of Kant´s transcendental philosophy based on the productive character of thought revealed by differential calculus.[104] For Maimon, the forms of thought are not applied to *given* appearances by means of schemata, as Kant claims. Rather, Maimon argues "that both the forms and the objects of our cognition themselves are in us *a priori*".[105] Our faculty of cognition does not consist merely

[99] Hermann Cohen, *Logik der reinen Erkenntnis. System Der Philosophie*, p. 156.

[100] See Hernán Pringe, "Dimitry Gawronsky: Reality and Actual Infinitesimals", in *Kant-Studien*, vol. 114, no. 1, 2023, pp 68–97.

[101] Hermann Cohen, *Einleitung mit kritischem Nachtrag zur "Geschichte des Materialismus"*, von F. A. Lange. 3. Aufl, 1914, reedited by H. Holzhey, Hildesheim, 1984, p. 89.

[102] For this reason, Cohen is critical of Lasswitz's "Denkmittel der Variabilität". See Kurd Lasswitz, *Geschichte der Atomistik*, 2 vol., Hamburg und Leipzig: Leopold Voss, 1890, vol. 1, p. 269ff. On Cassirer's reception of Cohen's views and the importance of this point, see Hernán Pringe, "Cohen's Logik der reinen Erkenntnis and Cassirer's Substanzbegriff und Funktionsbegriff", in *Kant Yearbook*, 2020, pp. 137–168.

[103] Natorp also identifies the differential quotient with the law of arising of the finite. For Natorp, however, the origin of the finite is the law, which can be conceived, intensively, to be concentrated in a point or, extensively, to be extended in the line. (Paul Natorp, *Die logischen Grundlagen der exakten Wissenschaften*, Teubner, Leipzig, 1910, p. 220) Bergman is too quick to identify Cohen's and Natorp's position in Samuel Bergman, *The Philosophy of Solomon Maimon* (translated by Noah H. Jacobs), Jerusalem: Magnes Press, 1967, p. 265.

[104] In his discussion of the relationship between Maimon and Cohen, Bergman seems to overlook the metaphysical implications of Cohen´s interpretation of differential calculus. See S. Bergman, *op. cit*., p. 270f.

[105] GW II: 432.

in recognizing given objects by means of forms of thought, but also "in producing the objects themselves by means of these forms."[106] Empirical objects are produced when the understanding thinks their differentials according to the categories. Accordingly, Maimon calls his position rational dogmatism and opposes it to Kant´s empirical dogmatism, which claims that the object of cognition must be given a posteriori, by experience.[107]

However, whereas we may fully determine the generating rule of a mathematical object, we can only progressively approach the rule in function of which the empirical arises. Clear and distinct consciousness of the generating rule of the empirical is possible only for an infinite understanding.[108] In the case of a limited understanding like our own, there is just the possibility of a progressive approximation to the consciousness of that rule.[109] This approximation is possible because the rule (i.e., the differential) is a representation of a spontaneity that differs from ours only quantitatively (and not qualitatively). But precisely because of this difference, the approximation never ends. In view of this infinite character of the path connecting the empirical and its supersensible ground, Maimon calls his position empirical skepticism, in opposition to Kant's rational skepticism, according to which there is *no* such a path *at all*.[110] Maimon's Leibnizian metaphysics is this infinite quest for the supersensible, towards which our limited intellect may draw ever closer in a never-ending progression.[111]

Whereas for Maimon the differential is an idea of an infinite understanding that our understanding can only approach asymptotically, for Cohen, the differential is the first product of thought. In *The Principle of the Infinitesimal Method and its History* and the second edition of *Kant's Theory of Experience*, Cohen applied the transcendental method to differential calculus and determined the conditions of its possibility.[112] This critique of knowledge revealed the productive character of thought and opened up the possibility of a metaphysics as a system of pure knowledge. The *Logic of Pure Knowledge* is the first part of this system. According

[106] GW II: 432.

[107] GW II: 432.

[108] Maimon's theory of differentials enables us to understand *how* this infinite understanding operates: "God creates the objects of nature in the same manner that we create the objects of mathematics: by real thought, i.e. by construction." GW IV: 58.

[109] As Bergman puts it, "God, as it were, thinks in differentials and we in integrals." (S. Bergman, *op. cit*., p. 63).

[110] GW II, 432. "To discover a passage from the sensible to the intelligible world", Maimon claims, whatever the politicians may say, "is certainly more important than the discovery of a route to the East Indies." GW II: 339.

[111] In the same sense, Bergman claims that a "metaphysics constructed on the basis of a consistent idealism in the manner of Leibniz is therefore *possible* (in contradistinction to Kant) since our intuitions keep approaching ideas just as in mathematics a series keeps approaching its limit." (S. Bergman, *op. cit*., p. 67).

[112] On Cohen's transcendental method, see Hernán Pringe, "Infinitesimal method and judgment of origin", *Kant E-prints*, serie 2, vol. 16 (2), 2021, pp. 185–199.

to Cohen, this logic (and not mere critique) of the infinitesimal, as the first and founding part of metaphysics, establishes the principles of pure thought by means of which it generates its own content. This logic of pure knowledge is the first element of a system that also includes pure will and pure feeling, which as pure reject any "external authority."[113] These three different expressions of spontaneity involve different kinds of laws and content, which must be preserved in their diversity, but nevertheless unified into a system. For Cohen, this unification is the specific task of psychology. Insofar as it is logic that establishes what such a unity is, logic remains the foundation of psychology and thus of the system of philosophy. But more than that, Cohen claims that it is only this unity that brings about the unity of man, and thus the concept of man. Therefore, Cohen's doctrine directs the entire system of philosophy toward the question of what man is.[114] Despite their profound differences, in this sense at least, the neo-Kantian Cohen remains faithful to Kant.[115]

[113] Hermann Cohen, *Einleitung mit kritischem Nachtrag zur "Geschichte des Materialismus"*, p. 44.

[114] Hermann Cohen, *Logik der reinen Erkenntnis. System Der Philosophie*, p. 520.

[115] "The field of philosophy in this cosmopolitan sense can be brought down to the following questions: 1. What can I know? 2. What ought I to do? 3. What may I hope? 4. What is man? Metaphysics answers the first question, morals the second, religion the third, and anthropology the fourth. Fundamentally, however, we could reckon all of this as anthropology, because the first three questions relate to the last one." (AA 9: 25).